

47. (a) We use the Bernoulli equation:  $p_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$ , where  $h_1$  is the height of the water in the tank,  $p_1$  is the pressure there, and  $v_1$  is the speed of the water there;  $h_2$  is the altitude of the hole,  $p_2$  is the pressure there, and  $v_2$  is the speed of the water there.  $\rho$  is the density of water. The pressure at the top of the tank and at the hole is atmospheric, so  $p_1 = p_2$ . Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then becomes  $\rho gh_1 = \frac{1}{2}\rho v_2^2 + \rho gh_2$  and

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.8 \text{ m/s}^2)(0.30 \text{ m})} = 2.42 \text{ m/s} .$$

The flow rate is  $A_2 v_2 = (6.5 \times 10^{-4} \text{ m}^2)(2.42 \text{ m/s}) = 1.6 \times 10^{-3} \text{ m}^3/\text{s}$ .

- (b) We use the equation of continuity:  $A_2 v_2 = A_3 v_3$ , where  $A_3 = \frac{1}{2}A_2$  and  $v_3$  is the water speed where the area of the stream is half its area at the hole. Thus  $v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84 \text{ m/s}$ . The water is in free fall and we wish to know how far it has fallen when its speed is doubled to  $4.84 \text{ m/s}$ . Since the pressure is the same throughout the fall,  $\frac{1}{2}\rho v_2^2 + \rho gh_2 = \frac{1}{2}\rho v_3^2 + \rho gh_3$ . Thus

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{(4.84 \text{ m/s})^2 - (2.42 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 0.90 \text{ m} .$$