

79. (a) We convert distances to meters, and use $v = \sqrt{GM/r}$ for speed when the probe is in circular orbit (this equation is readily obtained from Eq. 14-41). Our notations for the speeds are: v_o for the original speed of the probe when it is in a circular Venus-like orbit (of radius r_o); v_p for the speed when the rockets have fired and it is at the perihelion ($r_p = r_o$) of its subsequent elliptical orbit; and, v_f for its final speed once it is in a circular Earth-like orbit (of radius r_f which coincides with the aphelion distance r_a of the aforementioned ellipse). We find

$$v_o = \sqrt{\frac{GM}{r_o}} = \sqrt{\frac{(6.67 \times 10^{-11})(1.99 \times 10^{30})}{1.08 \times 10^{11}}} = 3.51 \times 10^4 \text{ m/s} .$$

With $m = 6000$ kg, the original energy is given by Eq. 14-44:

$$E_o = -\frac{GMm}{2r_o} = -3.69 \times 10^{12} \text{ J} .$$

Once the rockets have fired, the probe starts on an elliptical path with semimajor axis

$$a = \frac{r_p + r_a}{2} = \frac{r_o + r_f}{2} = 1.29 \times 10^{11} \text{ m}$$

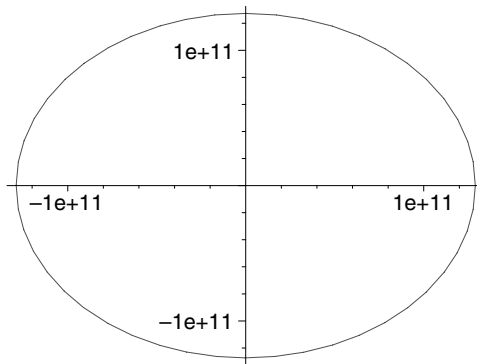
where $r_f = 1.5 \times 10^{11}$ m. By Eq. 14-46, its energy is now

$$E_{\text{ellipse}} = -\frac{GMm}{2a} = -3.09 \times 10^{12} \text{ J} .$$

The energy “boost” required when the probe is at r_o is therefore $E_{\text{ellipse}} - E_o = 6.0 \times 10^{11}$ J. The speed of the probe at the moment it has received this boost is figured from the kinetic energy ($v_p = \sqrt{2K/m}$) where $K = E_{\text{ellipse}} - U$. Thus,

$$v_p = \sqrt{\frac{2}{m} \left(-\frac{GMm}{2a} + \frac{GMm}{r_p} \right)} = 3.78 \times 10^4 \text{ m/s}$$

which means the speed increase is $v_p - v_o = 2.75 \times 10^3$ m/s. The orbit (if it were allowed to complete one full revolution) is plotted below. The Sun is not shown; it is not exactly at the center but rather 2.1×10^{10} m to the right of origin (if we are assuming the perihelion is the rightmost point shown and the aphelion is the leftmost point shown).



- (b) When the probe reaches $r_f = r_a$ it still has energy E_{ellipse} but now has speed

$$v_a = \frac{r_p v_p}{r_a} = \frac{(1.08 \times 10^{11})(3.78 \times 10^4)}{1.5 \times 10^{11}} = 2.722 \times 10^4 \text{ m/s}$$