

23. Using the fact that the volume of a sphere is $4\pi R^3/3$, we find the density of the sphere:

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{1.0 \times 10^4 \text{ kg}}{\frac{4}{3}\pi (1.0 \text{ m})^3} = 2.4 \times 10^3 \text{ kg/m}^3 .$$

When the particle of mass m (upon which the sphere, or parts of it, are exerting a gravitational force) is at radius r (measured from the center of the sphere), then whatever mass M is at a radius less than r must contribute to the magnitude of that force (GMm/r^2).

(a) At $r = 1.5 \text{ m}$, all of M_{total} is at a smaller radius and thus all contributes to the force:

$$|F_{\text{on } m}| = \frac{GmM_{\text{total}}}{r^2} = m (3.0 \times 10^{-7} \text{ N/kg}) .$$

(b) At $r = 0.50 \text{ m}$, the portion of the sphere at radius smaller than that is

$$M = \rho \left(\frac{4}{3}\pi r^3 \right) = 1.3 \times 10^3 \text{ kg} .$$

Thus, the force on m has magnitude $GMm/r^2 = m (3.3 \times 10^{-7} \text{ N/kg})$.

(c) Pursuing the calculation of part (b) algebraically, we find

$$|F_{\text{on } m}| = \frac{Gm\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = mr \left(6.7 \times 10^{-7} \frac{\text{N}}{\text{kg}\cdot\text{m}} \right) .$$