73. This problem involves the vector cross product of vectors lying in the xy plane. For such vectors, if we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}}$, then (using Eq. 3-30) we find

$$\vec{r}\,' \times \vec{v} = (x'v_y - y'v_x)\,\hat{\mathrm{k}}\ .$$

(a) Here, $\vec{r}\,'$ points in either the $+\hat{\mathrm{i}}$ or the $-\hat{\mathrm{i}}$ direction (since the particle moves along the x axis). It has no y' or z' components, and neither does $\vec{v}$, so it is clear from the above expression (or, more simply, from the fact that $\hat{\mathrm{i}} \times \hat{\mathrm{i}} = 0$) that $\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = 0$ in this case.

(b) The net force is in the $-\hat{\mathrm{i}}$ direction (as one finds from differentiating the velocity expression, yielding the acceleration), so, similar to what we found in part (a), we obtain $\tau = \vec{r}\,' \times \vec{F} = 0$.

(c) Now, $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = 2.0\,\hat{\mathrm{i}} + 5.0\,\hat{\mathrm{j}}$ (with SI units understood) and points from $(2.0, 5.0, 0)$ to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\,(\vec{r}\,' \times \vec{v}) = -m\,(\vec{r}_{\mathrm{o}} \times \vec{v}) = -(3.0)\,\big((2.0)(0) - (5.0)\left(-2.0t^3\right)\big)\,\hat{\mathrm{k}}$$

which yields $\vec{\ell} = -30t^3\,\hat{\mathrm{k}}$ in SI units (kg·m^2/s).

(d) The acceleration vector is given by $\vec{a} = \frac{d\vec{v}}{dt} = -6.0t^2\,\hat{\mathrm{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\,(\vec{r}\,' \times \vec{a}) = -m\,(\vec{r}_{\mathrm{o}} \times \vec{a}) = -(3.0)\,\big((2.0)(0) - (5.0)\left(-6.0t^2\right)\big)\,\hat{\mathrm{k}}$$

which yields $\vec{\tau} = -90t^2\,\hat{\mathrm{k}}$ in SI units (N·m).

(e) In this situation, $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = 2.0\,\hat{\mathrm{i}} - 5.0\,\hat{\mathrm{j}}$ (with SI units understood) and points from $(2.0, -5.0, 0)$ to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\,(\vec{r}\,' \times \vec{v}) = -m\,(\vec{r}_{\mathrm{o}} \times \vec{v}) = -(3.0)\,\big((2.0)(0) - (-5.0)\left(-2.0t^3\right)\big)\,\hat{\mathrm{k}}$$

which yields $\vec{\ell} = 30t^3\,\hat{\mathrm{k}}$ in SI units (kg·m^2/s).

(f) Again, the acceleration vector is given by $\vec{a} = -6.0t^2\,\hat{\mathrm{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\,(\vec{r}\,' \times \vec{a}) = -m\,(\vec{r}_{\mathrm{o}} \times \vec{a}) = -(3.0)\,\big((2.0)(0) - (-5.0)\left(-6.0t^2\right)\big)\,\hat{\mathrm{k}}$$

which yields $\vec{\tau} = 90t^2\,\hat{\mathrm{k}}$ in SI units (N·m).