64. Since we will be taking the vector cross product in the course of our calculations, below, we note first that when the two vectors in a cross product $\vec{A} \times \vec{B}$ are in the xy plane, we have $\vec{A} = A_x\hat{\text{i}} + A_y\hat{\text{j}}$ and $\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}}$, and Eq. 3-30 leads to

$$\vec{A} \times \vec{B} = (A_xB_y - A_yB_x)\,\hat{\text{k}}\ .$$

(a) We set up a coordinate system with its origin at the firing point, the positive x axis in the horizontal direction of motion of the projectile and the positive y axis vertically upward. The projectile moves in the xy plane, and if $+x$ is to our right then the "rotation" sense will be clockwise. Thus, we expect our answer to be negative. The position vector for the projectile (as a function of time) is given by

$$\vec{r} = (v_{0x}t)\hat{\text{i}} + \left(v_{0y}t - \frac{1}{2}gt^2\right)\hat{\text{j}} = (v_0\cos\theta_0 t)\hat{\text{i}} + (v_0\sin\theta_0 - gt)\hat{\text{j}}$$

and the velocity vector is

$$\vec{v} = v_x\hat{\text{i}} + v_y\hat{\text{j}} = (v_0\cos\theta_0)\,\hat{\text{i}} + (v_0\sin\theta_0 - gt)\hat{\text{j}}\ .$$

Thus (using the above observation about the cross product of vectors in the xy plane) the angular momentum of the projectile as a function of time is

$$\vec{\ell} = m\vec{r} \times \vec{v} = -\frac{1}{2}mv_0\cos\theta_0 gt^2\hat{\text{k}}\,.$$

(b) We take the derivative of our result in part (a): $\frac{d\vec{\ell}}{dt} = -v_0mgt\cos\theta_0\hat{\text{k}}$.

(c) Again using the above observation about the cross product of vectors in the xy plane, we find

$$\vec{r} \times \vec{F} = \left((v_0\cos\theta_0 t)\hat{\text{i}} + r_y\hat{\text{j}}\right) \times (-mg\hat{\text{j}}) = -v_0mgt\cos\theta_0\,\hat{\text{k}}$$

which is the same as the result in part (b).

(d) They are the same because $d\vec{\ell}/dt = \tau = \vec{r} \times \vec{F}$.