72. (a) Constant angular acceleration kinematics can be used to compute the angular acceleration  $\alpha$ . If  $\omega_0$  is the initial angular velocity and t is the time to come to rest, then

$$0 = \omega_0 + \alpha t \implies \alpha = -\frac{\omega_0}{t}$$

which yields -39/32 = -1.2 rev/s or (multiplying by  $2\pi$ ) -7.66 rad/s<sup>2</sup> for the value of  $\alpha$ .

(b) We use  $\tau = I\alpha$ , where  $\tau$  is the torque and I is the rotational inertia. The contribution of the rod to I is  $M\ell^2/12$  (Table 11-2(e)), where M is its mass and  $\ell$  is its length. The contribution of each ball is  $m(\ell/2)^2$ , where m is the mass of a ball. The total rotational inertia is

$$I = \frac{M\ell^2}{12} + 2\frac{m\ell^2}{4} = \frac{(6.40\,\mathrm{kg})(1.20\,\mathrm{m})^2}{12} + \frac{(1.06\,\mathrm{kg})(1.20\,\mathrm{m})^2}{2}$$

which yields  $I = 1.53 \text{ kg} \cdot \text{m}^2$ . The torque, therefore, is

$$\tau = (1.53 \,\mathrm{kg} \cdot \mathrm{m}^2) \left(-7.66 \,\mathrm{rad/s}^2\right) = -11.7 \,\,\mathrm{N} \cdot \mathrm{m}$$

(c) Since the system comes to rest the mechanical energy that is converted to thermal energy is simply the initial kinetic energy

$$K_i = \frac{1}{2} I \omega_0^2 = \frac{1}{2} (1.53 \text{ kg} \cdot \text{m}^2) ((2\pi)(39) \text{ rad/s})^2 = 4.59 \times 10^4 \text{ J}.$$

(d) We apply Eq. 11-13:

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = ((2\pi)(39) \, \text{rad/s}) \, (32.0 \, \text{s}) + \frac{1}{2} \left(-7.66 \, \text{rad/s}^2\right) (32.0 \, \text{s})^2$$

which yields 3920 rad or (dividing by  $2\pi$ ) 624 rev for the value of angular displacement  $\theta$ .

(e) Only the mechanical energy that is converted to thermal energy can still be computed without additional information. It is  $4.59 \times 10^4$  J no matter how  $\tau$  varies with time, as long as the system comes to rest.