30. The connection between angle θ (measured from vertical – see Fig. 8-29) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy) is given by $h = L(1 - \cos\theta)$ where L is the length of the pendulum.

(a) We use energy conservation in the form of Eq. 8-17.

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mgL\,(1 - \cos\theta_1) &= \frac{1}{2}mv_2^2 + mgL\,(1 - \cos\theta_2) \end{aligned}$$

This leads to

$$v_2 = \sqrt{2gL\,(\cos\theta_2 - \cos\theta_1)} = 1.4 \text{ m/s}$$

since $L = 1.4$ m, $\theta_1 = 30°$, and $\theta_2 = 20°$.

(b) The maximum speed v_3 is at the lowest point. Our formula for h gives $h_3 = 0$ when $\theta_3 = 0°$, as expected.

$$\begin{aligned} K_1 + U_1 &= K_3 + U_3 \\ 0 + mgL\,(1 - \cos\theta_1) &= \frac{1}{2}mv_3^2 + 0 \end{aligned}$$

This yields $v_3 = 1.9$ m/s.

(c) We look for an angle θ_4 such that the speed there is $v_4 = v_3/3$. To be as accurate as possible, we proceed algebraically (substituting $v_3^2 = 2gL\,(1 - \cos\theta_1)$ at the appropriate place) and plug numbers in at the end. Energy conservation leads to

$$\begin{aligned} K_1 + U_1 &= K_4 + U_4 \\ 0 + mgL\,(1 - \cos\theta_1) &= \frac{1}{2}mv_4^2 + mgL\,(1 - \cos\theta_4) \\ mgL\,(1 - \cos\theta_1) &= \frac{1}{2}m\,\frac{v_3^2}{9} + mgL\,(1 - \cos\theta_4) \\ -gL\cos\theta_1 &= \frac{1}{2}\,\frac{2gL\,(1 - \cos\theta_1)}{9} - gL\cos\theta_4 \end{aligned}$$

where in the last step we have subtracted out mgL and then divided by m. Thus, we obtain

$$\theta_4 = \cos^{-1}\left(\frac{1}{9} + \frac{8}{9}\cos\theta_1\right) = 28.2°$$

where we have quoted the answer to three significant figures since the problem gives θ_1 to three figures.