33. (a) Since  $\tau = dL/dt$ , the average torque acting during any interval  $\Delta t$  is given by  $\tau_{\text{avg}} = (L_f - L_i)/\Delta t$ , where  $L_i$  is the initial angular momentum and  $L_f$  is the final angular momentum. Thus

$$\tau_{\rm avg} = \frac{0.800 \, \rm kg \cdot m^2/s - 3.00 \, \rm kg \cdot m^2/s}{1.50 \, \rm s}$$

which yields  $\tau_{\text{avg}} = -1.467 \approx -1.47 \text{ N} \cdot \text{m}$ . In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) The angle turned is  $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ . If the angular acceleration  $\alpha$  is uniform, then so is the torque and  $\alpha = \tau/I$ . Furthermore,  $\omega_0 = L_i/I$ , and we obtain

$$\theta = \frac{L_i t + \frac{1}{2}\tau t^2}{I}$$
  
=  $\frac{(3.00 \text{ kg} \cdot \text{m}^2/\text{s})(1.50 \text{ s}) + \frac{1}{2}(-1.467 \text{ N} \cdot \text{m})(1.50 \text{ s})^2}{0.140 \text{ kg} \cdot \text{m}^2}$   
= 20.4 rad.

(c) The work done on the wheel is

$$W = \tau \theta = (-1.47 \,\mathrm{N \cdot m})(20.4 \,\mathrm{rad}) = -29.9 \,\mathrm{J}$$

where more precise values are used in the calculation than what is shown here. An equally good method for finding W is Eq. 11-44, which, if desired, can be rewritten as  $W = (L_f^2 - L_i^2)/2I$ .

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

$$P_{\rm avg} = -\frac{W}{\Delta t} = -\frac{-29.8\,{\rm J}}{1.50\,{\rm s}} = 19.9~{\rm W}~.$$