

53. The forces on the balloon are the force of gravity $m\vec{g}$ (down) and the force of the air \vec{F}_a (up). We take the $+y$ to be up, and use a to mean the *magnitude* of the acceleration (which is not its usual use in this chapter). When the mass is M (before the ballast is thrown out) the acceleration is downward and Newton's second law is $F_a - Mg = -Ma$. After the ballast is thrown out, the mass is $M - m$ (where m is the mass of the ballast) and the acceleration is upward. Newton's second law leads to $F_a - (M - m)g = (M - m)a$. The earlier equation gives $F_a = M(g - a)$, and this plugs into the new equation to give

$$M(g - a) - (M - m)g = (M - m)a \implies m = \frac{2Ma}{g + a} .$$