23. We consider $\vec{A}$ with (x, y) components given by $(A\cos\alpha, A\sin\alpha)$. Similarly, $\vec{B} \rightarrow (B\cos\beta, B\sin\beta)$. The angle (measured from the $+x$ direction) for their vector sum must have a slope given by

$$\tan\theta = \frac{A\sin\alpha + B\sin\beta}{A\cos\alpha + B\cos\beta} \ .$$

The problem requires that we now consider the orthogonal direction, where $\tan\theta + 90^\circ = -\cot\theta$. If this (the negative reciprocal of the above expression) is to equal the slope for their vector *difference*, then we must have

$$-\frac{A\cos\alpha + B\cos\beta}{A\sin\alpha + B\sin\beta} = \frac{A\sin\alpha - B\sin\beta}{A\cos\alpha - B\cos\beta} \ .$$

Multiplying both sides by $A\sin\alpha + B\sin\beta$ and doing the same with $A\cos\alpha - B\cos\beta$ yields

$$A^2\,\cos^2\alpha - B^2\,\cos^2\beta = A^2\,\sin^2\alpha - B^2\,\sin^2\beta \ .$$

Rearranging, using the $\cos^2\phi + \sin^2\phi = 1$ identity, we obtain

$$A^2 = B^2 \quad \Longrightarrow \quad A = B \ .$$

In a *later* section, the scalar (dot) product of vectors is presented and this result can be revisited with a more compact derivation.