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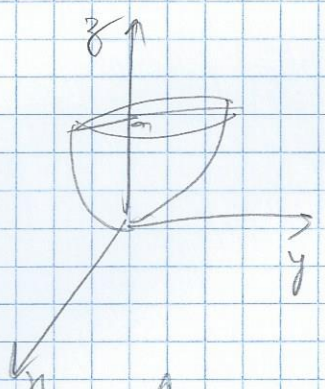
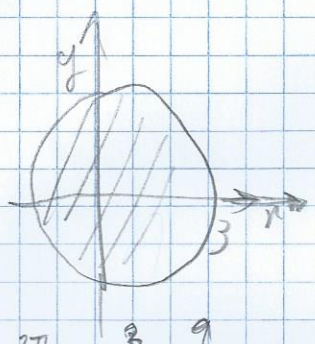
5) Calcule o volume do sólido compreendido entre $z = x^2 + y^2$ e o plano $z = 9$ (com coord. cilíndricas)



$$V = \iiint_D 1 \, dV \qquad 9 = x^2 + y^2$$

$$\begin{cases} x = R\cos\theta \\ y = R\operatorname{sen}\theta \\ z = z \end{cases} \qquad \begin{matrix} 0 \le \theta \le 2\pi \\ 0 \le R \le 3 \\ R^2 \le z \le 9 \end{matrix} \qquad |J| = R$$

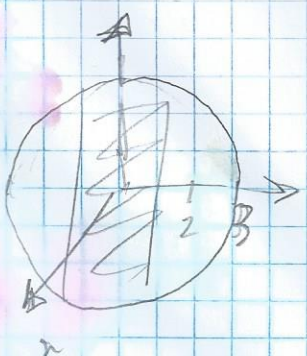


$$z = x^2 + y^2 \to z = R^2$$

$$V = \int_0^{2\pi} \int_0^3 \int_{R^2}^9 1 \cdot \underset{|J|}{R} \, dz \, dR \, d\theta = \int_0^{2\pi} \int_0^3 R\,[z]_{R^2}^9 \, dR \, d\theta =$$

$$= \int_0^{2\pi} \int_0^3 R\underbrace{(9 - R^2)}_{\text{Sup.}} \, dR \, d\theta = \int_0^{2\pi} \left[9\frac{R^2}{2} - \frac{R^4}{4} \right]_0^3 d\theta = \left(\frac{9}{2} 3^3 - \frac{3^4}{4} \right) 2\pi$$

6) Volume do sólido limitado sup. e inf. pela esfera $x^2 + y^2 + z^2 = 9$ e interior ao cilindro $x^2 + y^2 = 4$



$$\begin{cases} x = R\cos\theta \\ y = R\operatorname{sen}\theta \\ z = z \end{cases} \qquad \begin{matrix} 0 \le \theta \le 2\pi \\ 0 \le R \le 2 \end{matrix} \qquad |J| = R$$

$$-\sqrt{9 - R^2} \le z \le \sqrt{9 - R^2}$$

$$z^2 = 9 - (x^2 + y^2)$$
$$z = \pm\sqrt{9 - (x^2 + y^2)}$$