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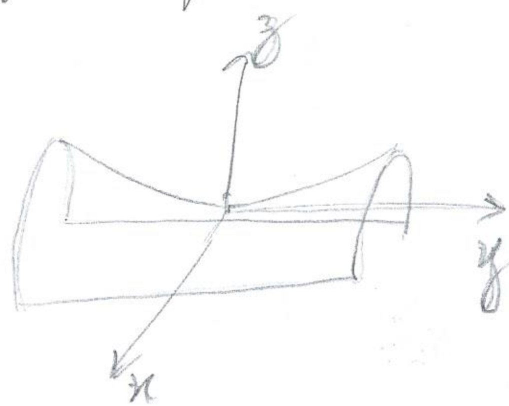
12) Det. o mín. e max. relativo e pontos sela de função $f(x,y) = xy - x^3 - y^2$

cálculo dos pontos críticos $\begin{cases} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \end{cases}$



$$\begin{cases} y - 3x^2 = 0 \\ x - 2y = 0 \end{cases} \Leftrightarrow \begin{cases} y = 3x^3 \\ x - 2.3x^2 = 0 \end{cases} \Rightarrow \begin{cases} - \\ x(1-6x) = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y = 0 \lor y = \frac{3}{36} = \frac{1}{2} \\ x = 0 \lor x = \frac{1}{6} \end{cases}$$

$(0,0)$ e $\left(\frac{1}{6}, \frac{1}{12}\right)$

Aplicar ao nosso caso

$A = \frac{\partial^2 f}{\partial x^2} = -6x$ $\quad B = \frac{\partial^2 f}{\partial y^2}$ $\quad C = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$

Teste da 2ª derivada

$D = A \times B - C^2 = \det \begin{bmatrix} A & C \\ C & B \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y \partial x} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$

$A = \frac{\partial^2 f}{\partial x^2} = -6x$ $\quad B = \frac{\partial^2 f}{\partial y^2} = -2$ $\quad C = \frac{\partial^2 f}{\partial y \partial x} = 1$

$D = A \times B - C^2$

Se $D > 0$ e $A > 0$ temos mín. relativo

Se $D > 0$ e $A < 0$ —"— máx. relativo

Se $D < 0$ —"— ponto sela

Se $D = 0$ nada podemos concluir.

... $(0,0)$ ponto sela