$$= \int_{-1}^{1} \left(x^2 y + \frac{y^3}{3} \right)_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} dx = \int_{-1}^{1} \Big[x^2 \left(1+\sqrt{1-x^2} - 1 + \sqrt{1-x^2}\right) +$$

$$+ \frac{\left(1+\sqrt{1-x^2}\right)^3}{3} - \left(\frac{\left(1-\sqrt{1-x^2}\right)^3}{3} \right) \Big] dx$$

T.P.C.
Em casa aplicar coord. polares & cap. 5 compl. ex.s

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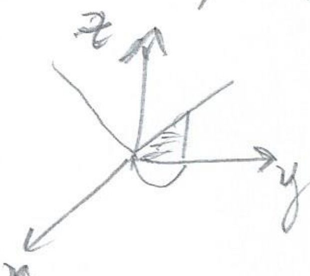
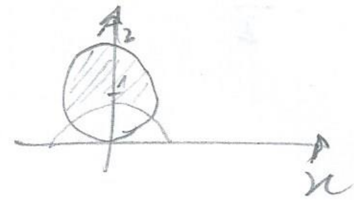
14) Volume (c/ coordenadas polares)? da superf. limitada superi. por $z=\sqrt{x^2+y^2}$ p/ dentro de $x^2+y^2=2y$ e inf. $z=0$



$x^2+(y-1)^2=1$



$$\begin{cases} x = R\cos\theta \\ y = R\,\text{sen}\,\theta \end{cases} \qquad \begin{array}{l} 0 \leq \theta \leq \pi \\ 0 \leq R \leq f(\theta) \\ \quad = 2\,\text{sen}\,\theta \end{array}$$

$x^2+y^2=2y$

$R^2 = 2R\,\text{sen}\,\theta \quad R=0 \vee R=2\cos(\theta)$

$$V = \int_0^{\pi} \int_0^{2\,\text{sen}\,\theta} \left(\sqrt{R^2} - 0\right) \left| \frac{\partial(x,y)}{\partial(R,\theta)} \right| dR\, d\theta =$$

$$= \int_0^{\pi} \int_0^{2\,\text{sen}\,\theta} (R \cdot \underbrace{R}_{|J|})\, dR\, d\theta = \int_0^{\pi} \left[\frac{R^3}{3} \right]_0^{2\,\text{sen}\,\theta} d\theta = \int_0^{\pi} \frac{8}{3}\,\text{sen}^3\theta\, d\theta =$$

$$= \frac{8}{3} \int_0^{\pi} \text{sen}^2\theta\, \text{sen}\,\theta\, d\theta = \frac{8}{3} \int_0^{\pi} (1-\cos^2\theta)\,\text{sen}\,\theta\, d\theta = \frac{8}{3} \int_0^{\pi} \left(\text{sen}\,\theta - \cos^2\theta\,\text{sen}\,\theta\right) d\theta$$

$$= \frac{8}{3} \left(-\cos\theta + \frac{\cos^3\theta}{3} \right)_0^{\pi} = \frac{32}{9}$$

$u = \cos\theta$
$u' = -\text{sen}\,\theta \quad P\, u'\, u^2$
$n = 2$