

Useando a variável

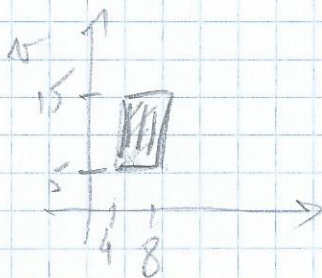
7) Calcule a área da região plana no 1º Q. limitada pelas curvas $xy=4$, $xy=8$, $xy^3=5$, $xy^3=15$

escolhendo a mudança de variáveis

$$\begin{cases} u=xy \\ v=xy^3 \end{cases}$$

$$4 \leq u \leq 8$$

$$5 \leq v \leq 15$$



$$A_R = \iint_R 1 dA = \int_4^8 \int_5^{15} 1 \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du dv$$

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \frac{1}{\left| \frac{\partial(u,v)}{\partial(x,y)} \right|} = \frac{1}{\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}} = \frac{1}{\begin{vmatrix} y & x \\ y^3 & 3xy^2 \end{vmatrix}} =$$

$$= \frac{1}{3xy^3 - xy^3} = \frac{1}{2xy^3} = \frac{1}{2v}$$

$$\int_4^8 \int_5^{15} 1 \cdot \frac{1}{2v} dv du = \int_4^8 \left[\frac{1}{2} (\ln v) \right]_5^{15} du =$$

$$= \int_4^8 \frac{1}{2} (\ln 15 - \ln 5) du = \frac{1}{2} \ln 3 [u]_4^8 = \ln 3$$