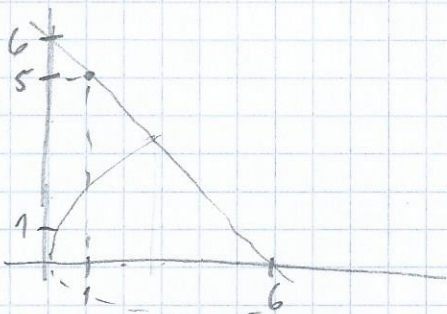


Integrais Duplas

Poj. 413-1033

19) $\iint_R xy \, dA$ R é a região compreendida entre $y = \sqrt{x}$

$$x = y^2$$



$$y = 6 - x \text{ e } y = 0$$

$$\begin{cases} y = \sqrt{x} \\ y = 6 - x \end{cases} \Rightarrow \begin{cases} \sqrt{x} = 6 - x \\ x = (6 - x)^2 \end{cases}$$

$$\begin{cases} x = 36 - 12x + x^2 \\ x^2 - 13x + 36 = 0 \end{cases}$$

$$x = \frac{13 \pm \sqrt{(13)^2 - 4(36)}}{2}$$

$$\begin{cases} x = 4 \vee x = 9 \\ y = 2 \end{cases}$$

$$\int_0^2 \left[\int_{y^2}^{6-y} xy \, dx \right] dy = \int_0^4 \left[\int_0^{\sqrt{x}} xy \, dy \right] dx + \int_4^6 \left[\int_0^{6-x} xy \, dy \right] dx$$

$$= \int_0^4 x \left[\frac{y^2}{2} \right]_0^{\sqrt{x}} dx + \int_4^6 x \left[\frac{y^2}{2} \right]_0^{6-x} dx = \int_0^4 x \frac{x}{2} dx + \int_4^6 \frac{x(6-x)^2}{2} dx$$

$$= \left[\frac{x^3}{2} \right]_0^4 + \int_4^6 \frac{x(36 - 12x + x^2)}{2} dx = \frac{4^3}{2} + \left[\frac{36x^2}{4} - \frac{6x^3}{3} + \frac{x^4}{2} \right]_4^6$$

$$= (2^2)^3 + \left[9 \cdot 6^2 - 2 \cdot 6^3 + \frac{6^4}{2} - 9 \cdot 4^2 + 2 \cdot 4^3 - \frac{4^4}{2} \right] =$$

$$= 2^5 + 9 \cdot 36 - 2 \cdot 2^3 \cdot 3^3 + \frac{2^{3y} \cdot 3^4}{2} - 9 \cdot 16 + 2 \cdot 64 - 2^3 = //$$