CHAPTER 3

PUBLIC-KEY CRYPTOGRAPHY AND MESSAGE AUTHENTICATION

Every Egyptian received two names, which were known respectively as the true name and the good name, or the great name and the little name; and while the good or little name was made public, the true or great name appears to have been carefully concealed.

—*The Golden Bough,* Sir James George Frazer

To guard against the baneful influence exerted by strangers is therefore an elementary dictate of savage prudence. Hence before strangers are allowed to enter a district, or at least before they are permitted to mingle freely with the inhabitants, certain ceremonies are often performed by the natives of the country for the purpose of disarming the strangers of their magical powers, or of disinfecting, so to speak, the tainted atmosphere by which they are supposed to be surrounded.

—*The Golden Bough*, Sir James George Frazer

In addition to message confidentiality, message authentication is an important network security function. This chapter examines three aspects of message authentication. First, we look at the use of message authentication codes and hash functions to provide message authentication. Then we look at public-key encryption principles and two specific public-key algorithms. These algorithms are useful in the exchange of conventional encryption keys. Then we look at the use of public-key encryption to produce digital signatures, which provides an enhanced form of message authentication.

3.1 APPROACHES TO MESSAGE AUTHENTICATION

Encryption protects against passive attack (eavesdropping). A different requirement is to protect against active attack (falsification of data and transactions). Protection against such attacks is known as message authentication.

A message, file, document, or other collection of data is said to be authentic when it is genuine and comes from its alleged source. Message authentication is a procedure that allows communicating parties to verify that received messages are authentic.[1] The two important aspects are to verify that the contents of the message have not been altered and that the source is authentic. We may also wish to verify a message's timeliness (it has not been artificially delayed and replayed) and sequence relative to other messages flowing between two parties. All of these concerns come under the category of data integrity as described in Chapter 1.

Authentication Using Conventional Encryption

It would seem possible to perform authentication simply by the use of symmetric encryption. If we assume that only the sender and receiver share a key (which is as it should be), then only the genuine sender would be able to encrypt a message

[1]For simplicity, for the remainder of this chapter, we refer to *message authentication.* By this we mean both authentication of transmitted messages and of stored data (*data authentication*).

successfully for the other participant, provided the receiver can recognize a valid message. Furthermore, if the message includes an error-detection code and a sequence number, the receiver is assured that no alterations have been made and that sequencing is proper. If the message also includes a timestamp, the receiver is assured that the message has not been delayed beyond that normally expected for network transit.

In fact, symmetric encryption alone is not a suitable tool for data authentication. To give one simple example, in the ECB mode of encryption, if an attacker reorders the blocks of ciphertext, then each block will still decrypt successfully. However, the reordering may alter the meaning of the overall data sequence. Although sequence numbers may be used at some level (e.g., each IP packet), it is typically not the case that a separate sequence number will be associated with each b-bit block of plaintext. Thus, block reordering is a threat.

Message Authentication without Message Encryption

In this section, we examine several approaches to message authentication that do not rely on encryption. In all of these approaches, an authentication tag is generated and appended to each message for transmission. The message itself is not encrypted and can be read at the destination independent of the authentication function at the destination.

Because the approaches discussed in this section do not encrypt the message, message confidentiality is not provided. As was mentioned, message encryption by itself does not provide a secure form of authentication. However, it is possible to combine authentication and confidentiality in a single algorithm by encrypting a message plus its authentication tag. Typically, however, message authentication is provided as a separate function from message encryption. [DAVI89] suggests three situations in which message authentication without confidentiality is preferable:

1. There are a number of applications in which the same message is broadcast to a number of destinations. Two examples are notification to users that the network is now unavailable and an alarm signal in a control center. It is cheaper and more reliable to have only one destination responsible for monitoring authenticity. Thus, the message must be broadcast in plaintext with an associated message authentication tag. The responsible system performs authentication. If a violation occurs, the other destination systems are alerted by a general alarm.
2. Another possible scenario is an exchange in which one side has a heavy load and cannot afford the time to decrypt all incoming messages. Authentication is carried out on a selective basis with messages being chosen at random for checking.
3. Authentication of a computer program in plaintext is an attractive service. The computer program can be executed without having to decrypt it every time, which would be wasteful of processor resources. However, if a message authentication tag were attached to the program, it could be checked whenever assurance is required of the integrity of the program.

Thus, there is a place for both authentication and encryption in meeting security requirements.

Message Authentication Code One authentication technique involves the use of a secret key to generate a small block of data, known as a **message authentication code (MAC)**, that is appended to the message. This technique assumes that two communicating parties, say A and B, share a common secret key K_{AB}. When A has a message to send to B, it calculates the message authentication code as a function of the message and the key: $\text{MAC}_M = \text{F}(K_{AB}, M)$. The message plus code are transmitted to the intended recipient. The recipient performs the same calculation on the received message, using the same secret key, to generate a new message authentication code. The received code is compared to the calculated code (Figure 3.1). If we assume that only the receiver and the sender know the identity of the secret key, and if the received code matches the calculated code, then the following statements apply:

1. The receiver is assured that the message has not been altered. If an attacker alters the message but does not alter the code, then the receiver's calculation of the code will differ from the received code. Because the attacker is assumed not to know the secret key, the attacker cannot alter the code to correspond to the alterations in the message.
2. The receiver is assured that the message is from the alleged sender. Because no one else knows the secret key, no one else could prepare a message with a proper code.
3. If the message includes a sequence number (such as is used with HDLC and TCP), then the receiver can be assured of the proper sequence, because an attacker cannot successfully alter the sequence number.

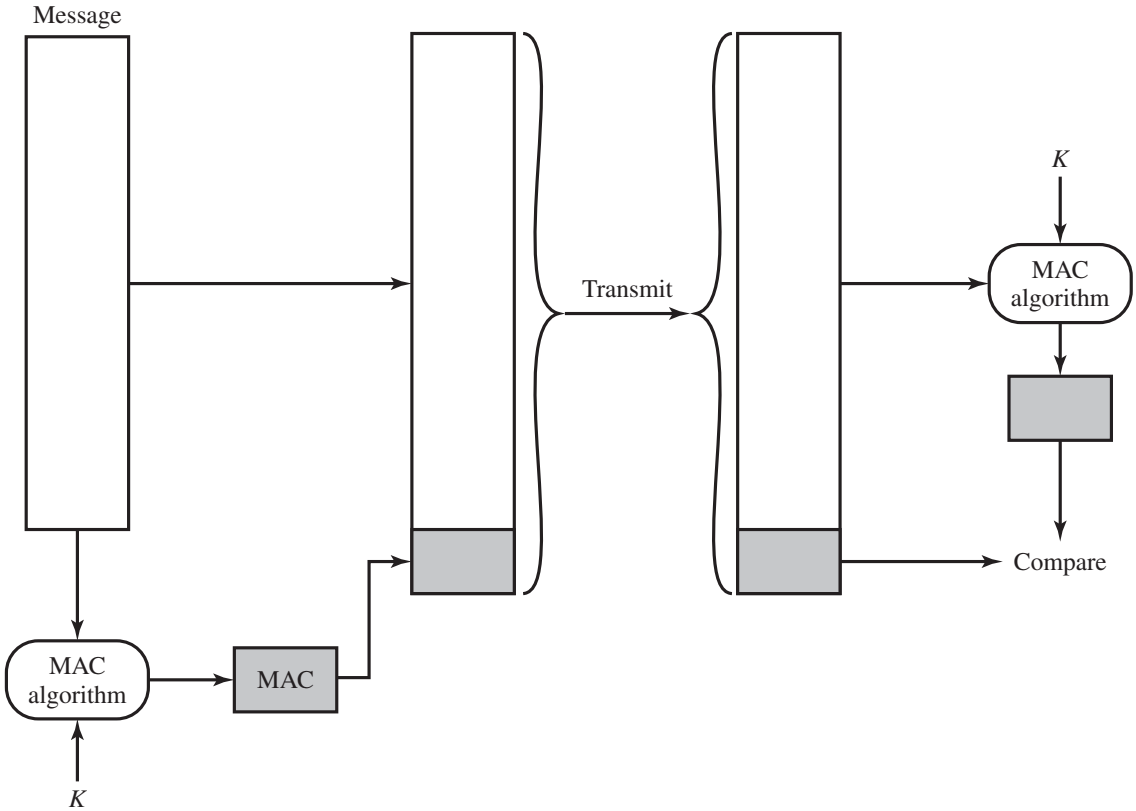


Figure 3.1 Message Authentication Using a Message Authentication Code (MAC)

A number of algorithms could be used to generate the code. The NIST specification, FIPS PUB 113, recommends the use of DES. DES is used to generate an encrypted version of the message, and the last number of bits of ciphertext are used as the code. A 16- or 32-bit code is typical.

The process just described is similar to encryption. One difference is that the authentication algorithm need not be reversible, as it must for decryption. Because of the mathematical properties of the authentication function, it is less vulnerable to being broken than encryption.

One-Way Hash Function An alternative to the message authentication code is the **one-way hash function**. As with the message authentication code, a hash function accepts a variable-size message M as input and produces a fixed-size message digest H(M) as output. Unlike the MAC, a hash function does not take a secret key as input. To authenticate a message, the message digest is sent with the message in such a way that the message digest is authentic.

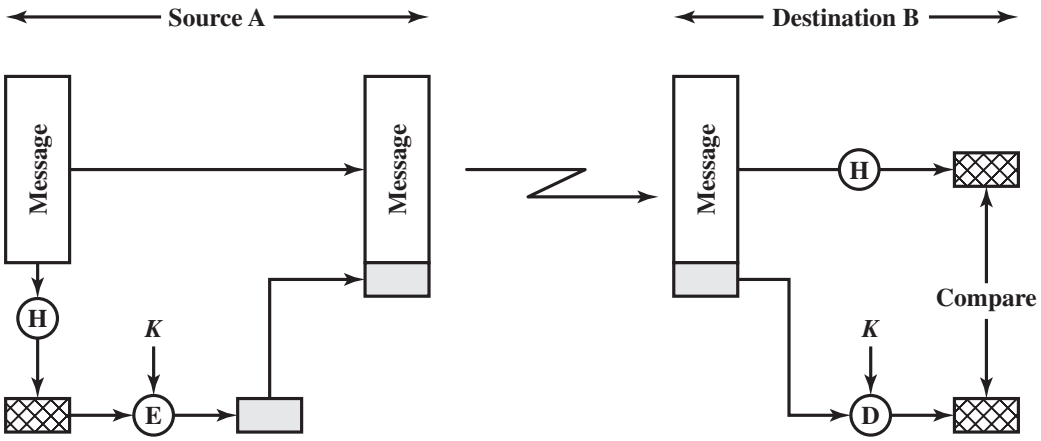
Figure 3.2 illustrates three ways in which the message can be authenticated. The message digest can be encrypted using conventional encryption (part a); if it is assumed that only the sender and receiver share the encryption key, then authenticity is assured. The message digest can be encrypted using public-key encryption (part b); this is explained in Section 3.5. The public-key approach has two advantages: (1) It provides a digital signature as well as message authentication. (2) It does not require the distribution of keys to communicating parties.

These two approaches also have an advantage over approaches that encrypt the entire message in that less computation is required. Nevertheless, there has been interest in developing a technique that avoids encryption altogether. Several reasons for this interest are pointed out in [TSUD92]:

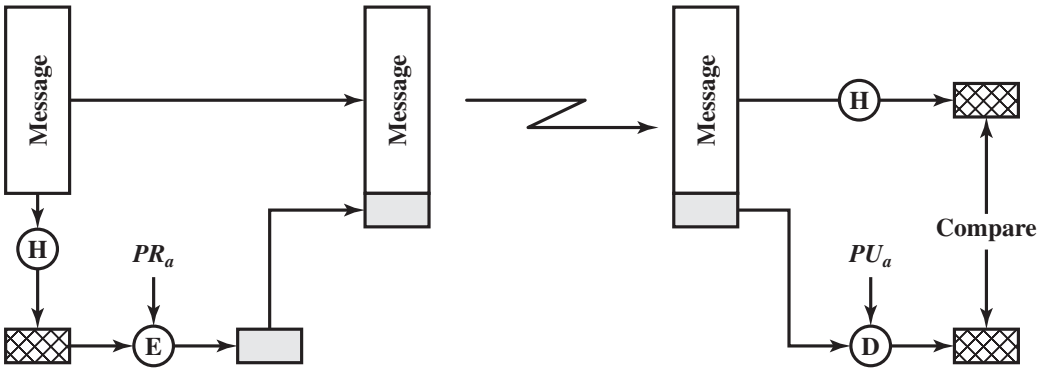
- Encryption software is quite slow. Even though the amount of data to be encrypted per message is small, there may be a steady stream of messages into and out of a system.
- Encryption hardware costs are nonnegligible. Low-cost chip implementations of DES are available, but the cost adds up if all nodes in a network must have this capability.
- Encryption hardware is optimized toward large data sizes. For small blocks of data, a high proportion of the time is spent in initialization/invocation overhead.
- An encryption algorithm may be protected by a patent.

Figure 3.2c shows a technique that uses a hash function but no encryption for message authentication. This technique assumes that two communicating parties, say A and B, share a common secret value S_{AB}. When A has a message to send to B, it calculates the hash function over the concatenation of the secret value and the message: $MD_M = \text{H}(S_{AB} \| M)$.[2] It then sends $[M \| MD_M]$ to B. Because B possesses S_{AB}, it can recompute $\text{H}(S_{AB} \| M)$ and verify MD_M. Because the secret value itself is

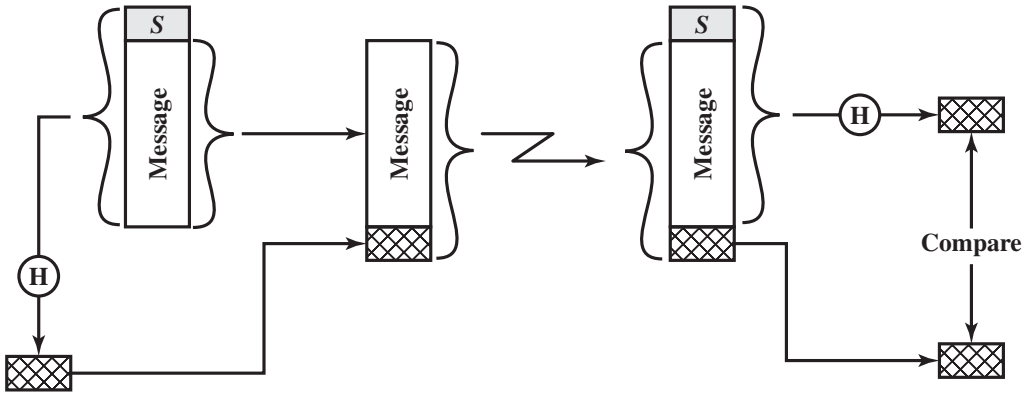
[2] $\|$ denotes concatenation.



(a) Using conventional encryption



(b) Using public-key encryption



(c) Using secret value

Figure 3.2 Message Authentication Using a One-Way Hash Function

not sent, it is not possible for an attacker to modify an intercepted message. As long as the secret value remains secret, it is also not possible for an attacker to generate a false message.

A variation on the third technique, called HMAC, is the one adopted for IP security (described in Chapter 8); it also has been specified for SNMPv3 (Chapter 12).

3.2 SECURE HASH FUNCTIONS

The one-way hash function, or **secure hash function**, is important not only in message authentication but in digital signatures. In this section, we begin with a discussion of requirements for a secure hash function. Then we look at the most important hash function, SHA.

Hash Function Requirements

The purpose of a hash function is to produce a "fingerprint" of a file, message, or other block of data. To be useful for message authentication, a hash function H must have the following properties:

1. H can be applied to a block of data of any size.
2. H produces a fixed-length output.
3. $H(x)$ is relatively easy to compute for any given x, making both hardware and software implementations practical.
4. For any given code h, it is computationally infeasible to find x such that $H(x) = h$. A hash function with this property is referred to as **one-way** or **preimage resistant**.[3]
5. For any given block x, it is computationally infeasible to find $y \neq x$ with $H(y) = H(x)$. A hash function with this property is referred to as **second preimage resistant**. This is sometimes referred to as **weak collision resistant**.
6. It is computationally infeasible to find any pair (x, y) such that $H(x) = H(y)$. A hash function with this property is referred to as **collision resistant**. This is sometimes referred to as **strong collision resistant**.

The first three properties are requirements for the practical application of a hash function to message authentication. The fourth property, preimage resistant, is the "one-way" property: It is easy to generate a code given a message, but virtually impossible to generate a message given a code. This property is important if the authentication technique involves the use of a secret value (Figure 3.2c). The secret value itself is not sent; however, if the hash function is not one way, an attacker can easily discover the secret value: If the attacker can observe or intercept a transmission, the attacker obtains the message M and the hash code $C = H(S_{AB} \| M)$. The attacker then inverts the hash function to obtain $S_{AB} \| M = H^{-1}(C)$. Because the attacker now has both M and $S_{AB} \| M$, it is a trivial matter to recover S_{AB}.

The second preimage resistant property guarantees that it is impossible to find an alternative message with the same hash value as a given message. This prevents forgery when an encrypted hash code is used (Figures 3.2a and b). If this property were not true, an attacker would be capable of the following sequence: First, observe or intercept a message plus its encrypted hash code; second, generate an unencrypted hash code from the message; third, generate an alternate message with the same hash code.

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[3]For $f(x) = y$, x is said to be a preimage of y. Unless f is one-to-one, there may be multiple preimage values for a given y.

A hash function that satisfies the first five properties in the preceding list is referred to as a weak hash function. If the sixth property is also satisfied, then it is referred to as a strong hash function. The sixth property, collision resistant, protects against a sophisticated class of attack known as the birthday attack. Details of this attack are beyond the scope of this book. The attack reduces the strength of an m-bit hash function from 2^m to $2^{m/2}$. See [STAL11] for details.

In addition to providing authentication, a message digest also provides data integrity. It performs the same function as a frame check sequence: If any bits in the message are accidentally altered in transit, the message digest will be in error.

Security of Hash Functions

As with symmetric encryption, there are two approaches to attacking a secure hash function: cryptanalysis and brute-force attack. As with symmetric encryption algorithms, cryptanalysis of a hash function involves exploiting logical weaknesses in the algorithm.

The strength of a hash function against brute-force attacks depends solely on the length of the hash code produced by the algorithm. For a hash code of length n, the level of effort required is proportional to the following:

Preimage resistant	2^n
Second preimage resistant	2^n
Collision resistant	$2^{n/2}$

If collision resistance is required (and this is desirable for a general-purpose secure hash code), then the value $2^{n/2}$ determines the strength of the hash code against brute-force attacks. Van Oorschot and Wiener [VANO94] presented a design for a $10 million collision search machine for MD5, which has a 128-bit hash length, that could find a collision in 24 days. Thus, a 128-bit code may be viewed as inadequate. The next step up, if a hash code is treated as a sequence of 32 bits, is a 160-bit hash length. With a hash length of 160 bits, the same search machine would require over four thousand years to find a collision. With today's technology, the time would be much shorter, so that 160 bits now appears suspect.

Simple Hash Functions

All hash functions operate using the following general principles. The input (message, file, etc.) is viewed as a sequence of n-bit blocks. The input is processed one block at a time in an iterative fashion to produce an n-bit hash function.

One of the simplest hash functions is the bit-by-bit exclusive-OR (XOR) of every block. This can be expressed as

$$C_i = b_{i1} \oplus b_{i2} \oplus \ldots \oplus b_{im}$$

where

C_i = ith bit of the hash code, $1 \le i \le n$
m = number of n-bit blocks in the input
b_{ij} = ith bit in jth block
$\oplus$ = XOR operation

	bit 1	bit 2	• • •	bit *n*
Block 1	b_{11}	b_{21}		b_{n1}
Block 2	b_{12}	b_{22}		b_{n2}
	• • •	• • •	• • •	• • •
Block *m*	b_{1m}	b_{2m}		b_{nm}
Hash code	C_1	C_2		C_n

Figure 3.3 Simple Hash Function Using Bitwise XOR

Figure 3.3 illustrates this operation; it produces a simple parity for each bit position and is known as a longitudinal redundancy check. It is reasonably effective for random data as a data integrity check. Each n-bit hash value is equally likely. Thus, the probability that a data error will result in an unchanged hash value is 2^{-n}. With more predictably formatted data, the function is less effective. For example, in most normal text files, the high-order bit of each octet is always zero. So if a 128-bit hash value is used, instead of an effectiveness of 2^{-128}, the hash function on this type of data has an effectiveness of 2^{-112}.

A simple way to improve matters is to perform a 1-bit circular shift, or rotation, on the hash value after each block is processed. The procedure can be summarized as

1. Initially set the n-bit hash value to zero.
2. Process each successive n-bit block of data:
   a. Rotate the current hash value to the left by one bit.
   b. XOR the block into the hash value.

This has the effect of "randomizing" the input more completely and overcoming any regularities that appear in the input.

Although the second procedure provides a good measure of data integrity, it is virtually useless for data security when an encrypted hash code is used with a plaintext message, as in Figures 3.2a and b. Given a message, it is an easy matter to produce a new message that yields that hash code: Simply prepare the desired alternate message and then append an n-bit block that forces the combined new message plus block to yield the desired hash code.

Although a simple XOR or rotated XOR (RXOR) is insufficient if only the hash code is encrypted, you may still feel that such a simple function could be useful when the message as well as the hash code are encrypted. But one must be careful. A technique originally proposed by the National Bureau of Standards used the simple XOR applied to 64-bit blocks of the message and then an encryption of the entire message using the cipher block chaining (CBC) mode. We can define the scheme as follows: Given a message consisting of a sequence of 64-bit blocks $X_1, X_2, \ldots, X_N$, define the hash code C as the block-by-block XOR or all blocks and append the hash code as the final block:

$$C = X_{N+1} = X_1 \oplus X_2 \oplus \ldots \oplus X_N$$

Next, encrypt the entire message plus hash code using CBC mode to produce the encrypted message $Y_1, Y_2, \ldots, Y_{N+1}$. [JUEN85] points out several ways in which the ciphertext of this message can be manipulated in such a way that it is not detectable by the hash code. For example, by the definition of CBC (Figure 2.10), we have

$$X_1 = IV \oplus \mathrm{D}(K, Y_1)$$
$$X_i = Y_{i-1} \oplus \mathrm{D}(K, Y_i)$$
$$X_{N+1} = \mathrm{Y}_N \oplus \mathrm{D}(K, Y_{N+1})$$

But X_{N+1} is the hash code:

$$\begin{aligned} X_{N+1} &= X_1 \oplus X_2 \oplus \ldots \oplus X_N \\ &= [IV \oplus \mathrm{D}(K, Y_1)] \oplus [Y_1 \oplus \mathrm{D}(K, Y_2)] \oplus \ldots \oplus [Y_{N-1} \oplus \mathrm{D}(K, Y_N)] \end{aligned}$$

Because the terms in the preceding equation can be XORed in any order, it follows that the hash code would not change if the ciphertext blocks were permuted.

The SHA Secure Hash Function

In recent years, the most widely used hash function has been the Secure Hash Algorithm (SHA). Indeed, because virtually every other widely used hash function had been found to have substantial cryptanalytic weaknesses, SHA was more or less the last remaining standardized hash algorithm by 2005. SHA was developed by the National Institute of Standards and Technology (NIST) and published as a federal information processing standard (FIPS 180) in 1993. When weaknesses were discovered in SHA (now known as SHA-0), a revised version was issued as FIPS 180-1 in 1995 and is referred to as **SHA-1**. The actual standards document is entitled "Secure Hash Standard." SHA is based on the hash function MD4, and its design closely models MD4. SHA-1 is also specified in RFC 3174, which essentially duplicates the material in FIPS 180-1 but adds a C code implementation.

SHA-1 produces a hash value of 160 bits. In 2002, NIST produced a revised version of the standard, FIPS 180-2, that defined three new versions of SHA with hash value lengths of 256, 384, and 512 bits known as SHA-256, SHA-384, and SHA-512, respectively. Collectively, these hash algorithms are known as **SHA-2**. These new versions have the same underlying structure and use the same types of modular arithmetic and logical binary operations as SHA-1. A revised document was issued as FIP PUB 180-3 in 2008, which added a 224-bit version (Table 3.1). SHA-2 is also specified in RFC 4634, which essentially duplicates the material in FIPS 180-3 but adds a C code implementation.

In 2005, NIST announced the intention to phase out approval of SHA-1 and move to a reliance on SHA-2 by 2010. Shortly thereafter, a research team described an attack in which two separate messages could be found that deliver the same SHA-1 hash using 2^{69} operations, far fewer than the 2^{80} operations previously thought needed to find a collision with an SHA-1 hash [WANG05]. This result should hasten the transition to SHA-2.

In this section, we provide a description of SHA-512. The other versions are quite similar.

Table 3.1 Comparison of SHA Parameters

	SHA-1	SHA-224	SHA-256	SHA-384	SHA-512
Message Digest Size	160	224	256	384	512
Message Size	$< 2^{64}$	$< 2^{64}$	$< 2^{64}$	$< 2^{128}$	$< 2^{128}$
Block Size	512	512	512	1024	1024
Word Size	32	32	32	64	64
Number of Steps	80	64	64	80	80
Security	80	112	128	192	256

Notes: 1. All sizes are measured in bits.
2. Security refers to the fact that a birthday attack on a message digest of size n produces a collision with a workfactor of approximately $2^{n/2}$.

The algorithm takes as input a message with a maximum length of less than 2^{128} bits and produces as output a 512-bit message digest. The input is processed in 1024-bit blocks. Figure 3.4 depicts the overall processing of a message to produce a digest. The processing consists of the following steps.

Step 1 **Append padding bits:** The message is padded so that its length is congruent to 896 modulo 1024 [length $\equiv$ 896 (mod 1024)]. Padding is always added, even if the message is already of the desired length. Thus, the number of padding bits is in the range of 1 to 1024. The padding consists of a single 1 bit followed by the necessary number of 0 bits.

Step 2 **Append length:** A block of 128 bits is appended to the message. This block is treated as an unsigned 128-bit integer (most significant byte first) and contains the length of the original message (before the padding).

The outcome of the first two steps yields a message that is an integer multiple of 1024 bits in length. In Figure 3.4, the expanded message is represented as the sequence of 1024-bit blocks $M_1, M_2, \ldots, M_N$, so that the total length of the expanded message is $N \times 1024$ bits.

Step 3 **Initialize hash buffer:** A 512-bit buffer is used to hold intermediate and final results of the hash function. The buffer can be represented as eight 64-bit registers (a, b, c, d, e, f, g, h). These registers are initialized to the following 64-bit integers (hexadecimal values):

$a =$ `6A09E667F3BCC908` $\quad e =$ `510E527FADE682D1`
$b =$ `BB67AE8584CAA73B` $\quad f =$ `9B05688C2B3E6C1F`
$c =$ `3C6EF372FE94F82B` $\quad g =$ `1F83D9ABFB41BD6B`
$d =$ `A54FF53A5F1D36F1` $\quad h =$ `5BE0CD19137E2179`

These values are stored in big-endian format, which is the most significant byte of a word in the low-address (leftmost) byte position. These words were obtained by taking the first sixty-four bits of the fractional parts of the square roots of the first eight prime numbers.

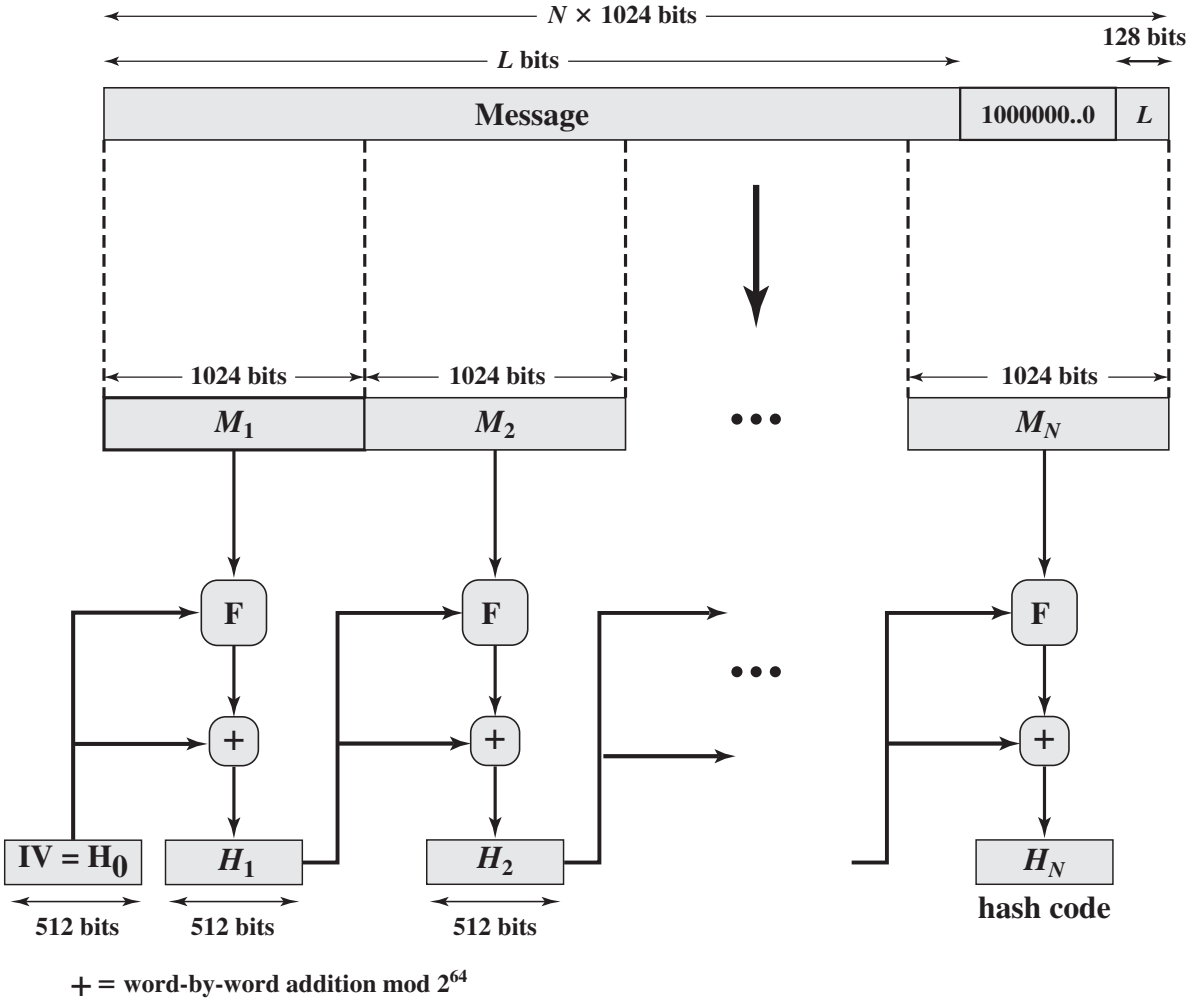


Figure 3.4 Message Digest Generation Using SHA-512

Step 4 **Process message in 1024-bit (128-word) blocks:** The heart of the algorithm is a module that consists of 80 rounds; this module is labeled F in Figure 3.4. The logic is illustrated in Figure 3.5.

Each round takes as input the 512-bit buffer value **`abcdefgh`** and updates the contents of the buffer. At input to the first round, the buffer has the value of the intermediate hash value, H_{i-1}. Each round t makes use of a 64-bit value W_t derived from the current 1024-bit block being processed (M_i). Each round also makes use of an additive constant K_t, where $0 \leq t \leq 79$ indicates one of the 80 rounds. These words represent the first 64 bits of the fractional parts of the cube roots of the first 80 prime numbers. The constants provide a "randomized" set of 64-bit patterns, which should eliminate any regularities in the input data.

The output of the 80th round is added to the input to the first round (H_{i-1}) to produce H_i. The addition is done independently for each of the eight words in the buffer with each of the corresponding words in H_{i-1}, using addition modulo 2^{64}.

Step 5 **Output:** After all N 1024-bit blocks have been processed, the output from the Nth stage is the 512-bit message digest.

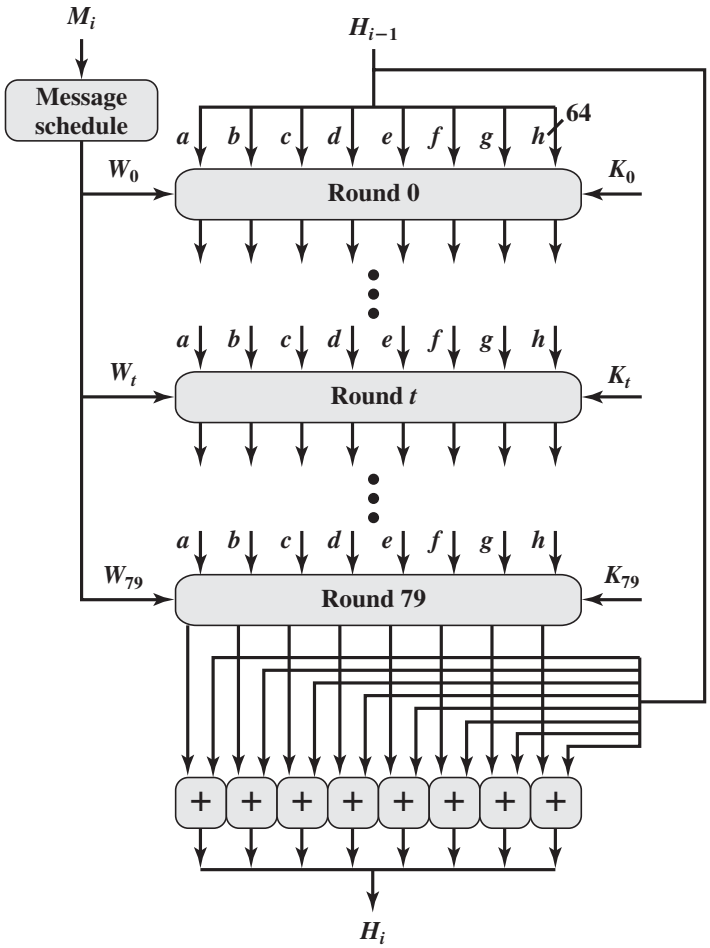


Figure 3.5 SHA-512 Processing of a Single 1024-Bit Block

The SHA-512 algorithm has the property that every bit of the hash code is a function of every bit of the input. The complex repetition of the basic function F produces results that are well mixed; that is, it is unlikely that two messages chosen at random, even if they exhibit similar regularities, will have the same hash code. Unless there is some hidden weakness in SHA-512, which has not so far been published, the difficulty of coming up with two messages having the same message digest is on the order of 2^{256} operations, while the difficulty of finding a message with a given digest is on the order of 2^{512} operations.

3.3 MESSAGE AUTHENTICATION CODES

HMAC

In recent years, there has been increased interest in developing a MAC derived from a cryptographic hash code, such as SHA-1. The motivations for this interest are

- Cryptographic hash functions generally execute faster in software than conventional encryption algorithms such as DES.
- Library code for cryptographic hash functions is widely available.

A hash function such as SHA-1 was not designed for use as a MAC and cannot be used directly for that purpose because it does not rely on a secret key. There have been a number of proposals for the incorporation of a secret key into an existing hash algorithm. The approach that has received the most support is HMAC [BELL96a, BELL96b]. HMAC has been issued as RFC 2104, has been chosen as the mandatory-to-implement MAC for IP Security, and is used in other Internet protocols, such as Transport Layer Security (TLS) and Secure Electronic Transaction (SET).

HMAC Design Objectives RFC 2104 lists the following design objectives for HMAC.

- To use, without modifications, available hash functions. In particular, hash functions that perform well in software, and for which code is freely and widely available
- To allow for easy replaceability of the embedded hash function in case faster or more secure hash functions are found or required
- To preserve the original performance of the hash function without incurring a significant degradation
- To use and handle keys in a simple way
- To have a well-understood cryptographic analysis of the strength of the authentication mechanism based on reasonable assumptions on the embedded hash function

The first two objectives are important to the acceptability of HMAC. HMAC treats the hash function as a "black box." This has two benefits. First, an existing implementation of a hash function can be used as a module in implementing HMAC. In this way, the bulk of the HMAC code is prepackaged and ready to use without modification. Second, if it is ever desired to replace a given hash function in an HMAC implementation, all that is required is to remove the existing hash function module and drop in the new module. This could be done if a faster hash function were desired. More important, if the security of the embedded hash function were compromised, the security of HMAC could be retained simply by replacing the embedded hash function with a more secure one.

The last design objective in the preceding list is, in fact, the main advantage of HMAC over other proposed hash-based schemes. HMAC can be proven secure provided that the embedded hash function has some reasonable cryptographic strengths. We return to this point later in this section, but first we examine the structure of HMAC.

HMAC Algorithm Figure 3.6 illustrates the overall operation of HMAC. The following terms are defined:

H = embedded hash function (e.g., SHA-1)

M = message input to HMAC (including the padding specified in the embedded hash function)

Y_i = ith block of M, $0 \le i \le (L - 1)$

L = number of blocks in M

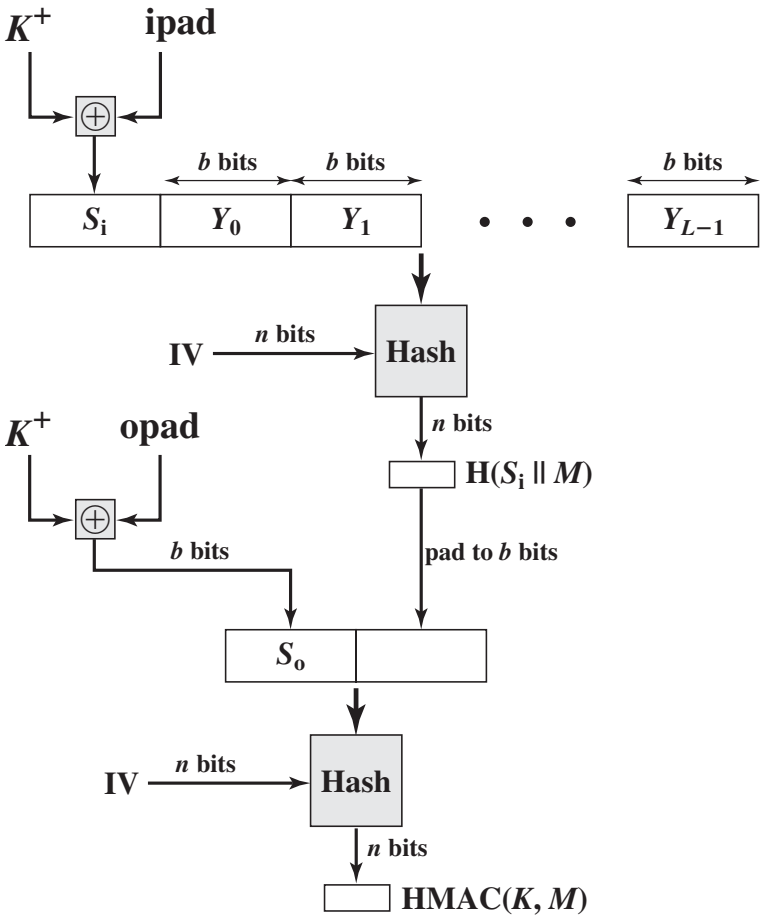


Figure 3.6 HMAC Structure

b = number of bits in a block

n = length of hash code produced by embedded hash function

K = secret key; if key length is greater than b, the key is input to the hash function to produce an n-bit key; recommended length is $> n$

K^+ = K padded with zeros on the left so that the result is b bits in length

ipad = 00110110 (36 in hexadecimal) repeated $b/8$ times

opad = 01011100 (5C in hexadecimal) repeated $b/8$ times

Then HMAC can be expressed as

$$\text{HMAC}(K, M) = \text{H}[(K^+ \oplus \text{opad}) \,\|\, \text{H}[(K^+ \oplus \text{ipad}) \,\|\, M]]$$

In words, HMAC is defined as follows:

1. Append zeros to the left end of K to create a b-bit string K^+ (e.g., if K is of length 160 bits and $b = 512$, then K will be appended with 44 zero bytes).
2. XOR (bitwise exclusive-OR) K^+ with ipad to produce the b-bit block S_i.

3. Append M to S_i.
4. Apply H to the stream generated in step 3.
5. XOR K^+ with opad to produce the b-bit block S_o.
6. Append the hash result from step 4 to S_o.
7. Apply H to the stream generated in step 6 and output the result.

Note that the XOR with ipad results in flipping one-half of the bits of K. Similarly, the XOR with opad results in flipping one-half of the bits of K, but a different set of bits. In effect, by passing S_i and S_o through the hash algorithm, we have pseudorandomly generated two keys from K.

HMAC should execute in approximately the same time as the embedded hash function for long messages. HMAC adds three executions of the basic hash function (for S_i, S_o, and the block produced from the inner hash).

MACs Based on Block Ciphers

In this section, we look at several MACs based on the use of a block cipher.

Cipher-Based Message Authentication Code (CMAC) The Cipher-based Message Authentication Code (CMAC) mode of operation is for use with AES and triple DES. It is specified in NIST Special Publication 800-38B.

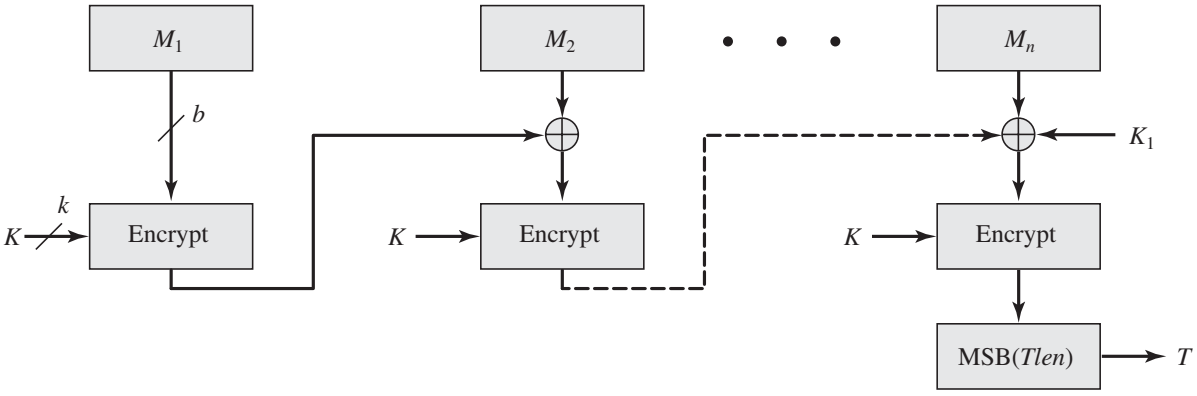
First, let us consider the operation of CMAC when the message is an integer multiple n of the cipher block length b. For AES, $b = 128$, and for triple DES, $b = 64$. The message is divided into n blocks $(M_1, M_2, \ldots, M_n)$. The algorithm makes use of a k-bit encryption key K and an n-bit key, K_1. For AES, the key size k is 128, 192, or 256 bits; for triple DES, the key size is 112 or 168 bits. CMAC is calculated as follows (Figure 3.7).

$$
\begin{aligned}
C_1 &= \mathrm{E}(K, M_1) \\
C_2 &= \mathrm{E}(K, [M_2 \oplus C_1]) \\
C_3 &= \mathrm{E}(K, [M_3 \oplus C_2]) \\
&\bullet \\
&\bullet \\
&\bullet \\
C_n &= \mathrm{E}(K, [M_N \oplus C_{n-1} \oplus K_1]) \\
T &= \mathrm{MSB}_{Tlen}(C_n)
\end{aligned}
$$

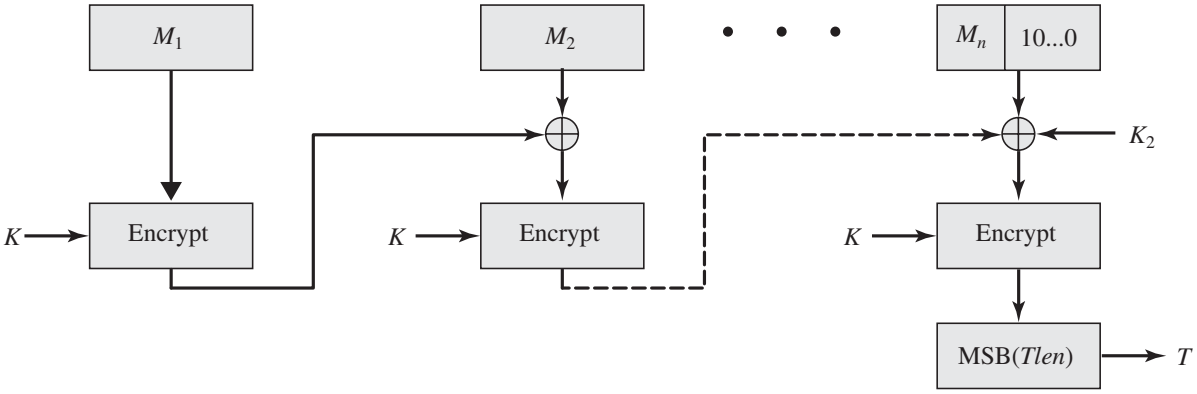
where

T = message authentication code, also referred to as the tag
$Tlen$ = bit length of T
$\mathrm{MSB}_s(X)$ = the s leftmost bits of the bit string X

If the message is not an integer multiple of the cipher block length, then the final block is padded to the right (least significant bits) with a 1 and as many 0s as necessary so that the final block is also of length b. The CMAC operation then proceeds as before, except that a different n-bit key K_2 is used instead of K_1.



(a) Message length is integer multiple of block size



(b) Message length is not integer multiple of block size

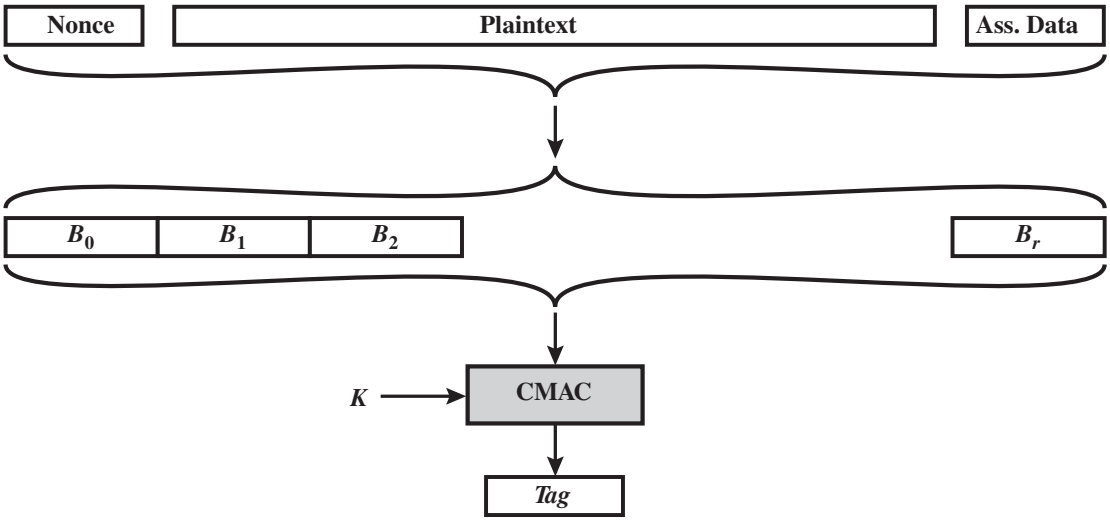
Figure 3.7 Cipher-Based Message Authentication Code (CMAC)

To generate the two n-bit keys, the block cipher is applied to the block that consists entirely of 0 bits. The first subkey is derived from the resulting ciphertext by a left shift of one bit and, conditionally, by XORing a constant that depends on the block size. The second subkey is derived in the same manner from the first subkey.

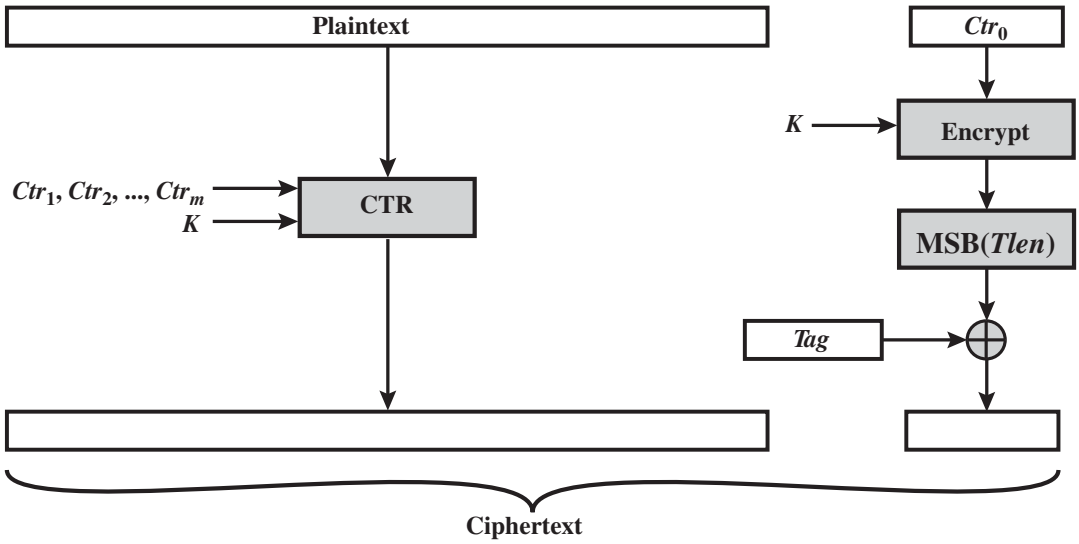
Counter with Cipher Block Chaining-Message Authentication Code The CCM mode of operation, defined in NIST SP 800-38C, is referred to as an **authenticated encryption** mode. Authenticated encryption is a term used to describe encryption systems that simultaneously protect confidentiality and authenticity (integrity) of communications. Many applications and protocols require both forms of security, but until recently the two services have been designed separately.

The key algorithmic ingredients of CCM are the AES encryption algorithm (Section 2.2), the CTR mode of operation (Section 2.5), and the CMAC authentication algorithm. A single key K is used for both encryption and MAC algorithms. The input to the CCM encryption process consists of three elements.

1. Data that will be both authenticated and encrypted. This is the plaintext message P of data block.
2. Associated data A that will be authenticated but not encrypted. An example is a protocol header that must be transmitted in the clear for proper protocol operation but which needs to be authenticated.



(a) Authentication



(b) Encryption

Figure 3.8 Counter with Cipher Block Chaining-Message Authentication Code (CCM)

3. A nonce N that is assigned to the payload and the associated data. This is a unique value that is different for every instance during the lifetime of a protocol association and is intended to prevent replay attacks and certain other types of attacks.

Figure 3.8 illustrates the operation of CCM. For authentication, the input includes the nonce, the associated data, and the plaintext. This input is formatted as a sequence of blocks B_0 through B_r. The first block contains the nonce plus some formatting bits that indicate the lengths of the N, A, and P elements. This is followed

by zero or more blocks that contain A, followed by zero of more blocks that contain P. The resulting sequence of blocks serves as input to the CMAC algorithm, which produces a MAC value with length *Tlen*, which is less than or equal to the block length (Figure 3.8a).

For encryption, a sequence of counters is generated that must be independent of the nonce. The authentication tag is encrypted in CTR mode using the single counter Ctr_0. The *Tlen* most significant bits of the output are XORed with the tag to produce an encrypted tag. The remaining counters are used for the CTR mode encryption of the plaintext (Figure 2.12). The encrypted plaintext is concatenated with the encrypted tag to form the ciphertext output (Figure 3.8b).

3.4 PUBLIC-KEY CRYPTOGRAPHY PRINCIPLES

Of equal importance to conventional encryption is **public-key encryption**, which finds use in message authentication and key distribution. This section looks first at the basic concept of public-key encryption and takes a preliminary look at key distribution issues. Section 3.5 examines the two most important public-key algorithms: RSA and Diffie-Hellman. Section 3.6 introduces digital signatures.

Public-Key Encryption Structure

Public-key encryption, first publicly proposed by Diffie and Hellman in 1976 [DIFF76], is the first truly revolutionary advance in encryption in literally thousands of years. Public-key algorithms are based on mathematical functions rather than on simple operations on bit patterns, such as are used in symmetric encryption algorithms. More important, public-key cryptography is asymmetric, involving the use of two separate keys—in contrast to the symmetric conventional encryption, which uses only one key. The use of two keys has profound consequences in the areas of confidentiality, key distribution, and authentication.

Before proceeding, we should first mention several common misconceptions concerning public-key encryption. One is that public-key encryption is more secure from cryptanalysis than conventional encryption. In fact, the security of any encryption scheme depends on (1) the length of the key and (2) the computational work involved in breaking a cipher. There is nothing in principle about either conventional or public-key encryption that makes one superior to another from the point of view of resisting cryptanalysis. A second misconception is that public-key encryption is a general-purpose technique that has made conventional encryption obsolete. On the contrary, because of the computational overhead of current public-key encryption schemes, there seems no foreseeable likelihood that conventional encryption will be abandoned. Finally, there is a feeling that key distribution is trivial when using public-key encryption, compared to the rather cumbersome handshaking involved with key distribution centers for conventional encryption. In fact, some form of protocol is needed, often involving a central agent, and the procedures involved are no simpler or any more efficient than those required for conventional encryption.

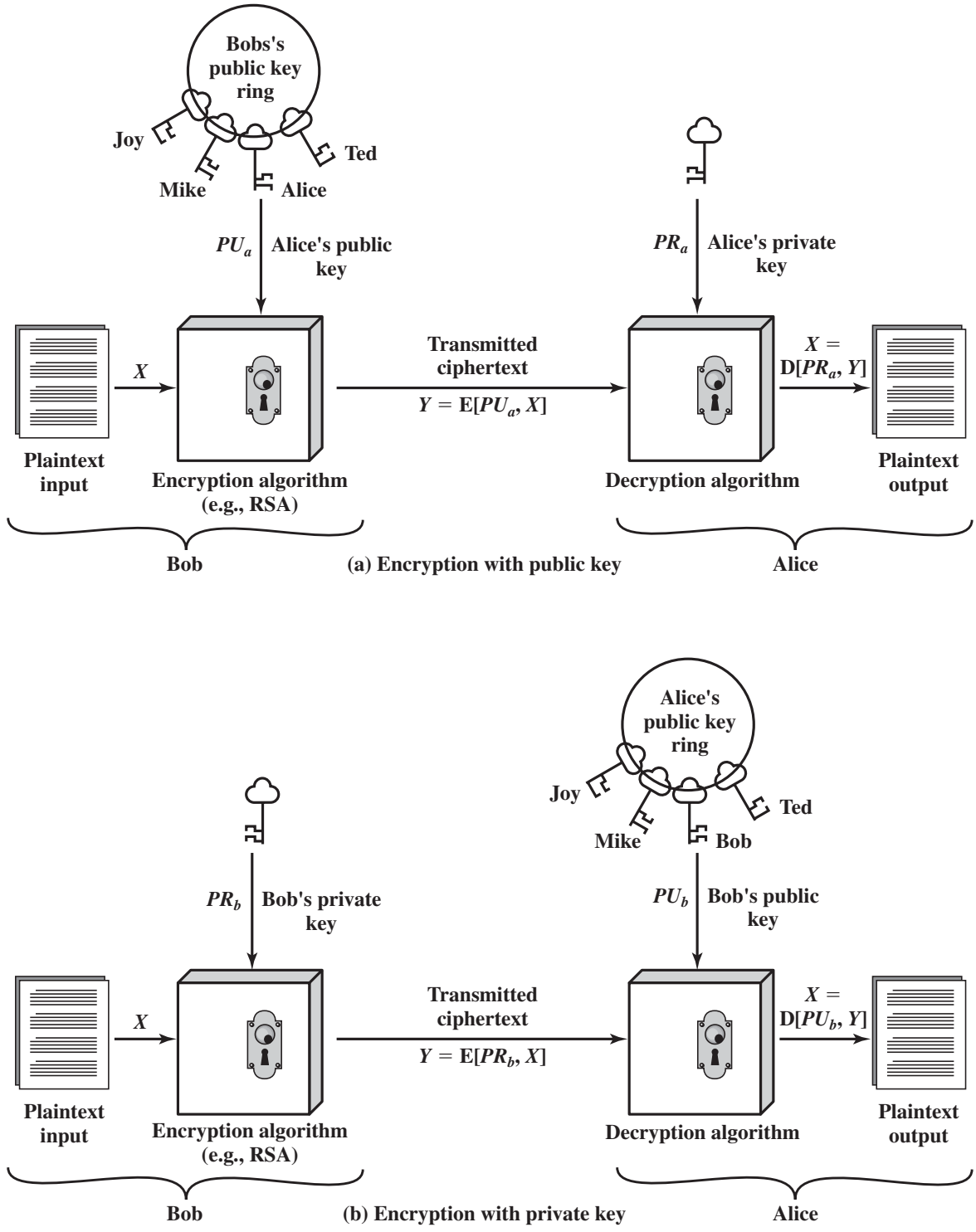


Figure 3.9 Public-Key Cryptography

A public-key encryption scheme has six ingredients (Figure 3.9a).

- **Plaintext:** This is the readable message or data that is fed into the algorithm as input.
- **Encryption algorithm:** The encryption algorithm performs various transformations on the plaintext.

- **Public and private key:** This is a pair of keys that have been selected so that if one is used for encryption, the other is used for decryption. The exact transformations performed by the encryption algorithm depend on the public or private key that is provided as input.
- **Ciphertext:** This is the scrambled message produced as output. It depends on the plaintext and the key. For a given message, two different keys will produce two different ciphertexts.
- **Decryption algorithm:** This algorithm accepts the ciphertext and the matching key and produces the original plaintext.

As the names suggest, the public key of the pair is made public for others to use, while the private key is known only to its owner. A general-purpose public-key cryptographic algorithm relies on one key for encryption and a different but related key for decryption.

The essential steps are the following:

1. Each user generates a pair of keys to be used for the encryption and decryption of messages.
2. Each user places one of the two keys in a public register or other accessible file. This is the public key. The companion key is kept private. As Figure 3.9a suggests, each user maintains a collection of public keys obtained from others.
3. If Bob wishes to send a private message to Alice, Bob encrypts the message using Alice's public key.
4. When Alice receives the message, she decrypts it using her private key. No other recipient can decrypt the message because only Alice knows Alice's private key.

With this approach, all participants have access to public keys, and private keys are generated locally by each participant and therefore need never be distributed. As long as a user protects his or her private key, incoming communication is secure. At any time, a user can change the private key and publish the companion public key to replace the old public key.

The key used in conventional encryption is typically referred to as a **secret key**. The two keys used for public-key encryption are referred to as the **public key** and the **private key**. Invariably, the private key is kept secret, but it is referred to as a private key rather than a secret key to avoid confusion with conventional encryption.

Applications for Public-Key Cryptosystems

Before proceeding, we need to clarify one aspect of public-key cryptosystems that is otherwise likely to lead to confusion. Public-key systems are characterized by the use of a cryptographic type of algorithm with two keys, one held private and one available publicly. Depending on the application, the sender uses either the sender's private key, the receiver's public key, or both to perform some type of cryptographic function. In broad terms, we can classify the use of public-key cryptosystems into three categories:

- **Encryption/decryption:** The sender encrypts a message with the recipient's public key.

Table 3.2 Applications for Public-Key Cryptosystems

Algorithm	Encryption/Decryption	Digital Signature	Key Exchange
RSA	Yes	Yes	Yes
Diffie-Hellman	No	No	Yes
DSS	No	Yes	No
Elliptic curve	Yes	Yes	Yes

- **Digital signature:** The sender "signs" a message with its private key. Signing is achieved by a cryptographic algorithm applied to the message or to a small block of data that is a function of the message.
- **Key exchange:** Two sides cooperate to exchange a session key. Several different approaches are possible, involving the private key(s) of one or both parties.

Some algorithms are suitable for all three applications, whereas others can be used only for one or two of these applications. Table 3.2 indicates the applications supported by the algorithms discussed in this chapter: RSA and Diffie Hellman. This table also includes the Digital Signature Standard (DSS) and elliptic-curve cryptography, also mentioned later in this chapter.

Requirements for Public-Key Cryptography

The cryptosystem illustrated in Figure 3.9 depends on a cryptographic algorithm based on two related keys. Diffie and Hellman postulated this system without demonstrating that such algorithms exist. However, they did lay out the conditions that such algorithms must fulfill [DIFF76]:

1. It is computationally easy for a party B to generate a pair (public key PU_b, private key PR_b).
2. It is computationally easy for a sender A, knowing the public key and the message to be encrypted, M, to generate the corresponding ciphertext:

$$C = \mathrm{E}(PU_b, M)$$

3. It is computationally easy for the receiver B to decrypt the resulting ciphertext using the private key to recover the original message:

$$M = \mathrm{D}(PR_b, C) = \mathrm{D}[PR_b, \mathrm{E}(PU_b, M)]$$

4. It is computationally infeasible for an opponent, knowing the public key, PU_b, to determine the private key, PR_b.
5. It is computationally infeasible for an opponent, knowing the public key, PU_b, and a ciphertext, C, to recover the original message, M.

We can add a sixth requirement that, although useful, is not necessary for all public-key applications.

6. Either of the two related keys can be used for encryption, with the other used for decryption.

$$M = \mathrm{D}[PU_b, \mathrm{E}(PR_b, M)] = \mathrm{D}[PR_b, \mathrm{E}(PU_b, M)]$$

3.5 PUBLIC-KEY CRYPTOGRAPHY ALGORITHMS

The two most widely used public-key algorithms are RSA and Diffie-Hellman. We look at both of these in this section and then briefly introduce two other algorithms.[4]

The RSA Public-Key Encryption Algorithm

One of the first public-key schemes was developed in 1977 by Ron Rivest, Adi Shamir, and Len Adleman at MIT and first published in 1978 [RIVE78]. The RSA scheme has since that time reigned supreme as the most widely accepted and implemented approach to public-key encryption. **RSA** is a block cipher in which the plaintext and ciphertext are integers between 0 and $n - 1$ for some n.

Encryption and decryption are of the following form period for some plaintext block M and ciphertext block C:

$$C = M^e \bmod n$$

$$M = C^d \bmod n = (M^e)^d \bmod n = M^{ed} \bmod n$$

Both sender and receiver must know the values of n and e, and only the receiver knows the value of d. This is a public-key encryption algorithm with a public key of $KU = \{e, n\}$ and a private key of $KR = \{d, n\}$. For this algorithm to be satisfactory for public-key encryption, the following requirements must be met.

1. It is possible to find values of e, d, n such that $M^{ed} \bmod n = M$ for all $M < n$.
2. It is relatively easy to calculate M^e and C^d for all values of $M < n$.
3. It is infeasible to determine d given e and n.

The first two requirements are easily met. The third requirement can be met for large values of e and n.

Figure 3.10 summarizes the RSA algorithm. Begin by selecting two prime numbers p and q and calculating their product n, which is the modulus for encryption and decryption. Next, we need the quantity $\phi(n)$, referred to as the Euler totient of n, which is the number of positive integers less than n and relatively prime to n. Then select an integer e that is relatively prime to $\phi(n)$ [i.e., the greatest common divisor of e and $\phi(n)$ is 1]. Finally, calculate d as the multiplicative inverse of e, modulo $\phi(n)$. It can be shown that d and e have the desired properties.

Suppose that user A has published its public key and that user B wishes to send the message M to A. Then B calculates $C = M^e \pmod n$ and transmits C. On receipt of this ciphertext, user A decrypts by calculating $M = C^d \pmod n$.

An example, from [SING99], is shown in Figure 3.11. For this example, the keys were generated as follows:

1. Select two prime numbers, $p = 17$ and $q = 11$.
2. Calculate $n = pq = 17 \times 11 = 187$.

[4]This section uses some elementary concepts from number theory. For a review, see Appendix A.

Key Generation

Select p, q	p and q both prime, $p \neq q$
Calculate $n = p \times q$	
Calculate $\phi(n) = (p - 1)(q - 1)$	
Select integer e	$\gcd(\phi(n), e) = 1; 1 < e < \phi(n)$
Calculate d	$de \bmod \phi(n) = 1$
Public key	$KU = \{e, n\}$
Private key	$KR = \{d, n\}$

Encryption

Plaintext:	$M < n$
Ciphertext:	$C = M^e \pmod{n}$

Decryption

Ciphertext:	C
Plaintext:	$M = C^d \pmod{n}$

Figure 3.10 The RSA Algorithm

3. Calculate $\phi(n) = (p - 1)(q - 1) = 16 \times 10 = 160$.
4. Select e such that e is relatively prime to $\phi(n) = 160$ and less than $\phi(n)$; we choose $e = 7$.
5. Determine d such that $de \bmod 160 = 1$ and $d < 160$. The correct value is $d = 23$, because $23 \times 7 = 161 = (1 \times 160) + 1$.

The resulting keys are public key $PU = \{7, 187\}$ and private key $PR = \{23, 187\}$. The example shows the use of these keys for a plaintext input of $M = 88$. For

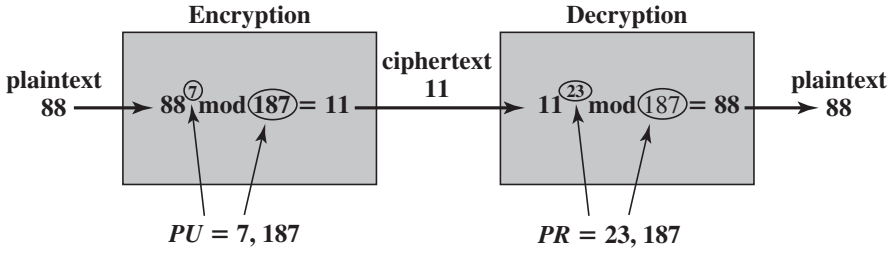


Figure 3.11 Example of RSA Algorithm

encryption, we need to calculate $C = 88^7 \bmod 187$. Exploiting the properties of modular arithmetic, we can do this as follows:

$$88^7 \bmod 187 = [(88^4 \bmod 187) \times (88^2 \bmod 187) \times (88^1 \bmod 187)] \bmod 187$$
$$88^1 \bmod 187 = 88$$
$$88^2 \bmod 187 = 7744 \bmod 187 = 77$$
$$88^4 \bmod 187 = 59{,}969{,}536 \bmod 187 = 132$$
$$88^7 \bmod 187 = (88 \times 77 \times 132) \bmod 187 = 894{,}432 \bmod 187 = 11$$

For decryption, we calculate $M = 11^{23} \bmod 187$:

$$11^{23} \bmod 187 = [(11^1 \bmod 187) \times (11^2 \bmod 187) \times (11^4 \bmod 187) \times (11^8 \bmod 187) \times (11^8 \bmod 187)] \bmod 187$$
$$11^1 \bmod 187 = 11$$
$$11^2 \bmod 187 = 121$$
$$11^4 \bmod 187 = 14{,}641 \bmod 187 = 55$$
$$11^8 \bmod 187 = 214{,}358{,}881 \bmod 187 = 33$$
$$11^{23} \bmod 187 = (11 \times 121 \times 55 \times 33 \times 33) \bmod 187 = 79{,}720{,}245 \bmod 187 = 88$$

There are two possible approaches to defeating the RSA algorithm. The first is the brute-force approach: Try all possible private keys. Thus, the larger the number of bits in e and d, the more secure the algorithm. However, because the calculations involved (both in key generation and in encryption/decryption) are complex, the larger the size of the key, the slower the system will run.

Most discussions of the cryptanalysis of RSA have focused on the task of factoring n into its two prime factors. For a large n with large prime factors, factoring is a hard problem, but not as hard as it used to be. A striking illustration of this occurred in 1977; the three inventors of RSA challenged *Scientific American* readers to decode a cipher they printed in Martin Gardner's "Mathematical Games" column [GARD77]. They offered a $100 reward for the return of a plaintext sentence, an event they predicted might not occur for some 40 quadrillion years. In April of 1994, a group working over the Internet and using over 1600 computers claimed the prize after only eight months of work [LEUT94]. This challenge used a public-key size (length of n) of 129 decimal digits (approximately 428 bits). This result does not invalidate the use of RSA; it simply means that larger key sizes must be used. Currently, a 1024-bit key size (about 300 decimal digits) is considered strong enough for virtually all applications.

Diffie–Hellman Key Exchange

The first published public-key algorithm appeared in the seminal paper by Diffie and Hellman that defined public-key cryptography [DIFF76] and is generally referred to as the **Diffie-Hellman key exchange**. A number of commercial products employ this key exchange technique.

The purpose of the algorithm is to enable two users to exchange a secret key securely that then can be used for subsequent encryption of messages. The algorithm itself is limited to the exchange of the keys.

The Diffie-Hellman algorithm depends for its effectiveness on the difficulty of computing discrete logarithms. Briefly, we can define the discrete logarithm in the following way. First, we define a primitive root of a prime number p as one whose powers generate all the integers from 1 to $p - 1$. That is, if a is a primitive root of the prime number p, then the numbers

$$a \bmod p, a^2 \bmod p, \ldots, ap^{-1} \bmod p$$

are distinct and consist of the integers from 1 through $p - 1$ in some permutation.

For any integer b less than p and a primitive root a of prime number p, one can find a unique exponent i such that

$$b = a^i \bmod p \quad 0 \leq i \leq (p - 1)$$

The exponent i is referred to as the discrete logarithm, or index, of b for the base a, mod p. We denote this value as $\text{dlog}_{a,p}(b)$.[5]

The Algorithm With this background, we can define the Diffie-Hellman key exchange, which is summarized in Figure 3.12. For this scheme, there are two publicly known numbers: a prime number q and an integer α that is a primitive root of q. Suppose the users A and B wish to exchange a key. User A selects a random integer $X_A < q$ and computes $Y_A = \alpha^{X_A} \bmod q$. Similarly, user B independently selects a random integer $X_B < q$ and computes $Y_B = \alpha^{X_B} \bmod q$. Each side keeps the X value private and makes the Y value available publicly to the other side. User A computes the key as $K = (Y_B)^{X_A} \bmod q$ and user B computes the key as $K = (Y_A)^{X_B} \bmod q$. These two calculations produce identical results:

$$\begin{aligned} K &= (Y_B)^{X_A} \bmod q \\ &= (\alpha^{X_B} \bmod q)^{X_A} \bmod q \\ &= (\alpha^{X_B})^{X_A} \bmod q \\ &= \alpha^{X_B X_A} \bmod q \\ &= (\alpha^{X_A})^{X_B} \bmod q \\ &= (\alpha^{X_A} \bmod q)^{X_B} \bmod q \\ &= (Y_A)^{X_B} \bmod q \end{aligned}$$

The result is that the two sides have exchanged a secret value. Furthermore, because X_A and X_B are private, an adversary only has the following ingredients to work with: q, α, Y_A, and Y_B. Thus, the adversary is forced to take a discrete logarithm to determine the key. For example, to determine the private key of user B, an adversary must compute

$$X_B = \text{dlog}_{\alpha,q}(Y_B)$$

[5]Many texts refer to the discrete logarithm as the *index*. There is no generally agreed notation for this concept, much less an agreed name.

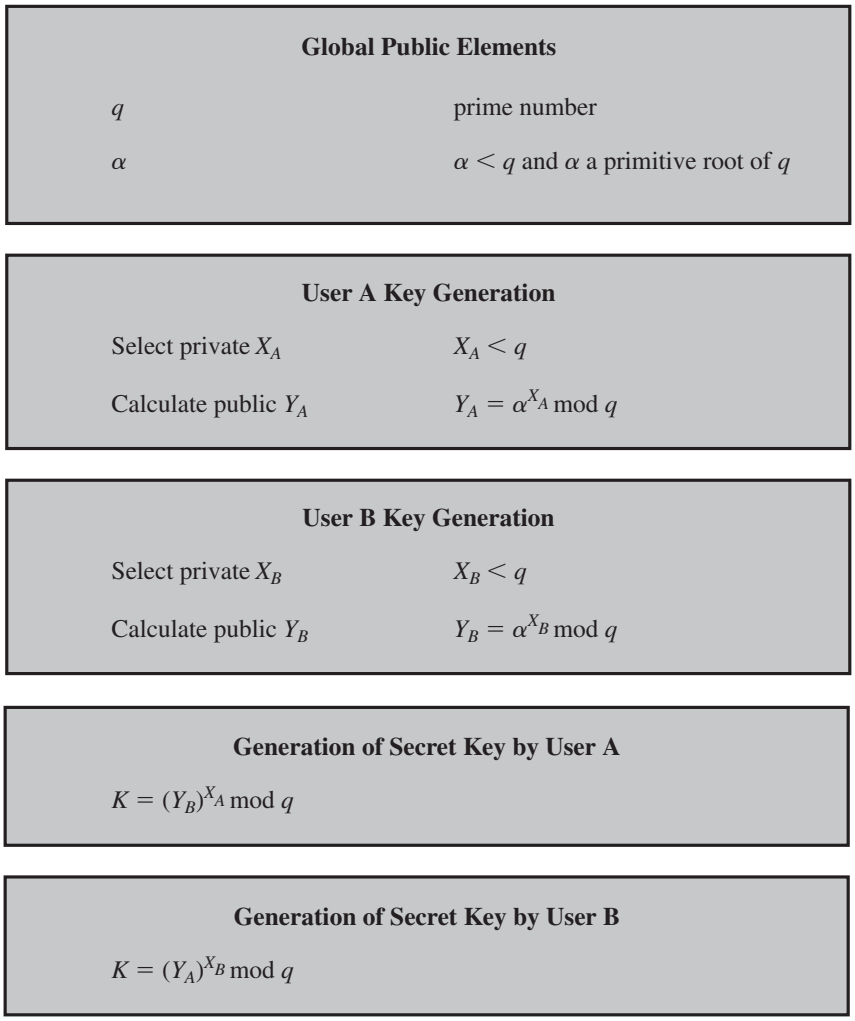


Figure 3.12 The Diffie-Hellman Key Exchange Algorithm

The adversary can then calculate the key K in the same manner as user B does.

The security of the Diffie-Hellman key exchange lies in the fact that, while it is relatively easy to calculate exponentials modulo a prime, it is very difficult to calculate discrete logarithms. For large primes, the latter task is considered infeasible.

Here is an example. Key exchange is based on the use of the prime number $q = 353$ and a primitive root of 353, in this case $\alpha = 3$. A and B select secret keys $X_A = 97$ and $X_B = 233$, respectively. Each computes its public key:

$$\text{A computes } Y_A = 3^{97} \bmod 353 = 40.$$
$$\text{B computes } Y_B = 3^{233} \bmod 353 = 248.$$

After they exchange public keys, each can compute the common secret key:

$$\text{A computes } K = (Y_B)^{X_A} \bmod 353 = 248^{97} \bmod 353 = 160.$$
$$\text{B computes } K = (Y_A)^{X_B} \bmod 353 = 40^{233} \bmod 353 = 160.$$

We assume an attacker would have available the following information:

$$q = 353; \quad \alpha = 3; \quad Y_A = 40; \quad Y_B = 248$$

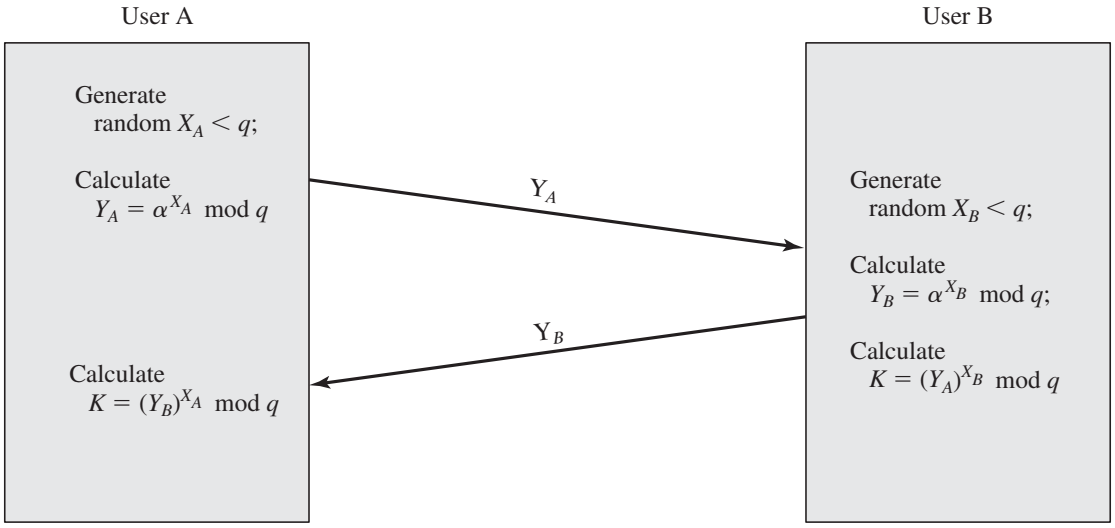


Figure 3.13 Diffie-Hellman Key Exchange

In this simple example, it would be possible to determine the secret key 160 by brute force. In particular, an attacker E can determine the common key by discovering a solution to the equation $3^a \bmod 353 = 40$ or the equation $3^b \bmod 353 = 248$. The brute-force approach is to calculate powers of 3 modulo 353, stopping when the result equals either 40 or 248. The desired answer is reached with the exponent value of 97, which provides $3^{97} \bmod 353 = 40$.

With larger numbers, the problem becomes impractical.

Key Exchange Protocols Figure 3.13 shows a simple protocol that makes use of the Diffie-Hellman calculation. Suppose that user A wishes to set up a connection with user B and use a secret key to encrypt messages on that connection. User A can generate a one-time private key X_A, calculate Y_A, and send that to user B. User B responds by generating a private value X_B, calculating Y_B, and sending Y_B to user A. Both users can now calculate the key. The necessary public values q and α would need to be known ahead of time. Alternatively, user A could pick values for q and α and include those in the first message.

As an example of another use of the Diffie-Hellman algorithm, suppose that a group of users (e.g., all users on a LAN) each generate a long-lasting private value X_A and calculate a public value Y_A. These public values, together with global public values for q and α, are stored in some central directory. At any time, user B can access user A's public value, calculate a secret key, and use that to send an encrypted message to user A. If the central directory is trusted, then this form of communication provides both confidentiality and a degree of authentication. Because only A and B can determine the key, no other user can read the message (confidentiality). Recipient A knows that only user B could have created a message using this key (authentication). However, the technique does not protect against replay attacks.

Man-in-the-Middle Attack The protocol depicted in Figure 3.13 is insecure against a man-in-the-middle attack. Suppose Alice and Bob wish to exchange keys, and Darth is the adversary. The attack proceeds as follows:

1. Darth prepares for the attack by generating two random private keys X_{D1} and X_{D2}, and then computing the corresponding public keys Y_{D1} and Y_{D2}.

2. Alice transmits Y_A to Bob.
3. Darth intercepts Y_A and transmits Y_{D1} to Bob. Darth also calculates $K2 = (Y_A)^{X_{D2}} \bmod q$.
4. Bob receives Y_{D1} and calculates $K1 = (Y_{D1})^{X_B} \bmod q$.
5. Bob transmits Y_B to Alice.
6. Darth intercepts Y_B and transmits Y_{D2} to Alice. Darth calculates $K1 = (Y_B)^{X_{D1}} \bmod q$.
7. Alice receives Y_{D2} and calculates $K2 = (Y_{D2})^{X_A} \bmod q$.

At this point, Bob and Alice think that they share a secret key. Instead Bob and Darth share secret key $K1$, and Alice and Darth share secret key $K2$. All future communication between Bob and Alice is compromised in the following way:

1. Alice sends an encrypted message M: E($K2$, M).
2. Darth intercepts the encrypted message and decrypts it to recover M.
3. Darth sends Bob E($K1$, M) or E($K1$, M'), where M' is any message. In the first case, Darth simply wants to eavesdrop on the communication without altering it. In the second case, Darth wants to modify the message going to Bob.

The key exchange protocol is vulnerable to such an attack because it does not authenticate the participants. This vulnerability can be overcome with the use of digital signatures and public-key certificates; these topics are explored later in this chapter and in Chapter 4.

Other Public-Key Cryptography Algorithms

Two other public-key algorithms have found commercial acceptance: DSS and elliptic-curve cryptography.

Digital Signature Standard The National Institute of Standards and Technology (NIST) has published Federal Information Processing Standard FIPS PUB 186, known as the **Digital Signature Standard (DSS)**. The DSS makes use of the SHA-1 and presents a new digital signature technique, the Digital Signature Algorithm (DSA). The DSS was originally proposed in 1991 and revised in 1993 in response to public feedback concerning the security of the scheme. There was a further minor revision in 1996. The DSS uses an algorithm that is designed to provide only the digital signature function. Unlike RSA, it cannot be used for encryption or key exchange.

Elliptic-Curve Cryptography The vast majority of the products and standards that use public-key cryptography for encryption and digital signatures use RSA. The bit length for secure RSA use has increased over recent years, and this has put a heavier processing load on applications using RSA. This burden has ramifications, especially for electronic commerce sites that conduct large numbers of secure transactions. Recently, a competing system has begun to challenge RSA: **elliptic curve cryptography (ECC)**. Already, ECC is showing up in standardization efforts, including the IEEE P1363 Standard for Public-Key Cryptography.

The principal attraction of ECC compared to RSA is that it appears to offer equal security for a far smaller bit size, thereby reducing processing overhead. On

the other hand, although the theory of ECC has been around for some time, it is only recently that products have begun to appear and that there has been sustained cryptanalytic interest in probing for weaknesses. Thus, the confidence level in ECC is not yet as high as that in RSA.

ECC is fundamentally more difficult to explain than either RSA or Diffie-Hellman, and a full mathematical description is beyond the scope of this book. The technique is based on the use of a mathematical construct known as the elliptic curve.

3.6 DIGITAL SIGNATURES

Public-key encryption can be used in another way, as illustrated in Figure 3.9b. Suppose that Bob wants to send a message to Alice, and although it is not important that the message be kept secret, he wants Alice to be certain that the message is indeed from him. In this case, Bob uses his own private key to encrypt the message. When Alice receives the ciphertext, she finds that she can decrypt it with Bob's public key, thus proving that the message must have been encrypted by Bob. No one else has Bob's private key, and therefore no one else could have created a ciphertext that could be decrypted with Bob's public key. Therefore, the entire encrypted message serves as a **digital signature**. In addition, it is impossible to alter the message without access to Bob's private key, so the message is authenticated both in terms of source and in terms of data integrity.

In the preceding scheme, the entire message is encrypted. Although validating both author and contents, this requires a great deal of storage. Each document must be kept in plaintext to be used for practical purposes. A copy also must be stored in ciphertext so that the origin and contents can be verified in case of a dispute. A more efficient way of achieving the same results is to encrypt a small block of bits that is a function of the document. Such a block, called an authenticator, must have the property that it is infeasible to change the document without changing the authenticator. If the authenticator is encrypted with the sender's private key, it serves as a signature that verifies origin, content, and sequencing. A secure hash code such as SHA-1 can serve this function. Figure 3.2b illustrates this scenario.

It is important to emphasize that the encryption process just described does not provide confidentiality. That is, the message being sent is safe from alteration but not safe from eavesdropping. This is obvious in the case of a signature based on a portion of the message, because the rest of the message is transmitted in the clear. Even in the case of complete encryption, there is no protection of confidentiality because any observer can decrypt the message by using the sender's public key.

3.7 RECOMMENDED READING AND WEB SITES

Solid treatments of hash functions and message authentication codes are found in [STIN06] and {MENE97].

The recommended treatments of encryption provided in Chapter 2 cover public-key as well as conventional encryption. [DIFF88] describes in detail the several attempts to devise secure two-key cryptoalgorithms and the gradual evolution of a variety of protocols based on them.

DIFF88 Diffie, W. "The First Ten Years of Public-Key Cryptography." *Procedings of the IEEE,* May 1988.

MENE97 Menezes, A.; Oorschot, P.; and Vanstone, S. *Handbook of Applied Cryptography.* Boca Raton, FL: CRC Press, 1997.

STIN06 Stinson, D. *Cryptography: Theory and Practice.* Boca Raton, FL: Chapman&Hall/ CRC Press, 2006.

Recommended Web Sites:

- **NIST Secure Hashing Page:** SHA FIPS and related documents.
- **RSA Laboratories:** Extensive collection of technical material on RSA and other topics in cryptography.
- **Digital Signatures:** NIST page with information on NIST-approved digital signature options.

3.8 KEY TERMS, REVIEW QUESTIONS, AND PROBLEMS

Key Terms

authenticated encryption	MD5	public-key encryption
Diffie-Hellman key exchange	message authentication	RIPEMD-160
digital signature	message authentication code (MAC)	RSA
Digital Signature Standard (DSS)	message digest	secret key
elliptic-curve cryptography (ECC)	one-way hash function	secure hash function
HMAC	private key	SHA-1
key exchange	public key	strong collision resistance
	public-key certificate	weak collision resistance

Review Questions

3.1 List three approaches to message authentication.
3.2 What is a message authentication code?
3.3 Briefly describe the three schemes illustrated in Figure 3.2.
3.4 What properties must a hash function have to be useful for message authentication?
3.5 In the context of a hash function, what is a compression function?
3.6 What are the principal ingredients of a public-key cryptosystem?
3.7 List and briefly define three uses of a public-key cryptosystem.

3.8 What is the difference between a private key and a secret key?

3.9 What is a digital signature?

Problems

3.1 Consider a 32-bit hash function defined as the concatenation of two 16-bit functions: XOR and RXOR, which are defined in Section 3.2 as "two simple hash functions."

a. Will this checksum detect all errors caused by an odd number of error bits? Explain.
b. Will this checksum detect all errors caused by an even number of error bits? If not, characterize the error patterns that will cause the checksum to fail.
c. Comment on the effectiveness of this function for use as a hash function for authentication.

3.2 Suppose H(m) is a collision-resistant hash function that maps a message of arbitrary bit length into an n-bit hash value. Is it true that, for all messages x, x' with $x \neq x'$, we have H(x) $\neq$ H(x')? Explain your answer.

3.3 State the value of the padding field in SHA-512 if the length of the message is

a. 1919 bits
b. 1920 bits
c. 1921 bits

3.4 State the value of the length field in SHA-512 if the length of the message is

a. 1919 bits
b. 1920 bits
c. 1921 bits

3.5 **a.** Consider the following hash function. Messages are in the form of a sequence of decimal numbers, $M = (a_1, a_2, \ldots, a_t)$. The hash value h is calculated as $\left(\sum_{i=1}^{t} a_i\right) \bmod n$, for some predefined value n. Does this hash function satisfy any of the requirements for a hash function listed in Section 3.2? Explain your answer.

b. Repeat part (a) for the hash function $h = \left(\sum_{i=1}^{t} (a_i)^2\right) \bmod n$.

c. Calculate the hash function of part (b) for $M = (189, 632, 900, 722, 349)$ and $n = 989$.

3.6 This problem introduces a hash function similar in spirit to SHA that operates on letters instead of binary data. It is called the *toy tetragraph hash* (tth).[6] Given a message consisting of a sequence of letters, tth produces a hash value consisting of four letters. First, tth divides the message into blocks of 16 letters, ignoring spaces, punctuation, and capitalization. If the message length is not divisible by 16, it is padded out with nulls. A four-number running total is maintained that starts out with the value (0, 0, 0, 0); this is input to the compression function for processing the first block. The compression function consists of two rounds. **Round 1:** Get the next block of text and arrange it as a row-wise 4×4 block of text and covert it to numbers (A = 0, B = 1, etc.). For example, for the block ABCDEFGHIJKLMNOP, we have

A	B	C	D
E	F	G	H
I	J	K	L
M	N	O	P

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

[6] I thank William K. Mason of the magazine staff of *The Cryptogram* for providing this example.

Then, add each column mod 26 and add the result to the running total, mod 26. In this example, the running total is (24, 2, 6, 10). **Round 2:** Using the matrix from round 1, rotate the first row left by 1, second row left by 2, third row left by 3, and reverse the order of the fourth row. In our example:

B	C	D	A
G	H	E	F
L	I	J	K
P	O	N	M

1	2	3	0
6	7	4	5
11	8	9	10
15	14	13	12

Now, add each column mod 26 and add the result to the running total. The new running total is (5, 7, 9, 11). This running total is now the input into the first round of the compression function for the next block of text. After the final block is processed, convert the final running total to letters. For example, if the message is ABCDE FGHIJKLMNOP, then the hash is FHJL.

a. Draw figures comparable to Figures 3.4 and 3.5 to depict the overall tth logic and the compression function logic.
b. Calculate the hash function for the 48-letter message "I leave twenty million dollars to my friendly cousin Bill."
c. To demonstrate the weakness of tth, find a 48-letter block that produces the same hash as that just derived. *Hint:* Use lots of A's.

3.7 It is possible to use a hash function to construct a block cipher with a structure similar to DES. Because a hash function is one way and a block cipher must be reversible (to decrypt), how is it possible?

3.8 Now consider the opposite problem: Use an encryption algorithm to construct a one-way hash function. Consider using RSA with a known key. Then process a message consisting of a sequence of blocks as follows: Encrypt the first block, XOR the result with the second block and encrypt again, and so on. Show that this scheme is not secure by solving the following problem. Given a two-block message B1, B2, and its hash, we have

$$\text{RSAH}(\text{B1}, \text{B2}) = \text{RSA}(\text{RSA}(\text{B1}) \oplus \text{B2})$$

Given an arbitrary block C1, choose C2 so that RSAH(C1, C2) = RSAH(B1, B2). Thus, the hash function does not satisfy weak collision resistance.

3.9 One of the most widely used MACs, referred to as the Data Authentication Algorithm, is based on DES. The algorithm is both a FIPS publication (FIPS PUB 113) and an ANSI standard (X9.17). The algorithm can be defined as using the cipher block chaining (CBC) mode of operation of DES with an initialization vector of zero (Figure 2.10). The data (e.g., message, record, file, or program) to be authenticated is grouped into contiguous 64-bit blocks: $P_1, P_2, \ldots, P_N$. If necessary, the final block is padded on the right with 0s to form a full 64-bit block. The MAC consists of either the entire ciphertext block C_N or the leftmost M bits of the block with $16 \leq M \leq 64$. Show that the same result can be produced using the cipher feedback mode.

3.10 In this problem, we will compare the security services that are provided by digital signatures (DS) and message authentication codes (MAC). We assume that Oscar is able to observe all messages send from Alice to Bob and vice versa. Oscar has no knowledge of any keys but the public one in case of DS. State whether and how (i) DS and (ii) MAC protect against each attack. The value `auth(x)` is computed with a DS or a MAC algorithm, respectively.

a. (Message integrity) Alice sends a message `x = "Transfer $1000 to Mark"` in the clear and also sends `auth(x)` to Bob. Oscar intercepts the message and replaces "Mark" with "Oscar". Will Bob detect this?

b. (Replay) Alice sends a message `x = "Transfer $1000 to Oscar"` in the clear and also sends `auth(x)` to Bob. Oscar observes the message and signature and sends them 100 times to Bob. Will Bob detect this?

c. (Sender Authentication with cheating third party) Oscar claims that he sent some message `x` with a valid `auth(x)` to Bob, but Alice claims the same. Can Bob clear the question in either case?

d. (Authentication with Bob cheating) Bob claims that he received a message `x` with a valid signature `auth(x)` from Alice (e.g., "Transfer $1000 from Alice to Bob") but Alice claims she has never sent it. Can Alice clear this question in either case?

3.11 Figure 3.14 shows an alternative means of implementing HMAC.

a. Describe the operation of this implementation.

b. What potential benefit does this implementation have over that shown in Figure 3.6?

3.12 In this problem, we demonstrate that for CMAC, a variant that XORs the second key after applying the final encryption doesn't work. Let us consider this for the case of the message being an integer multiple of the block size. Then the variant can be expressed as $\text{VMAC}(K, M) = \text{CBC}(K, M) \oplus K_1$. Now suppose an adversary is able to ask for the MACs of three messages: the message $\mathbf{0} = 0^n$, where n is the cipher block size; the message $\mathbf{1} = 1^n$; and the message $\mathbf{1} \| \mathbf{0}$. As a result of these three queries, the

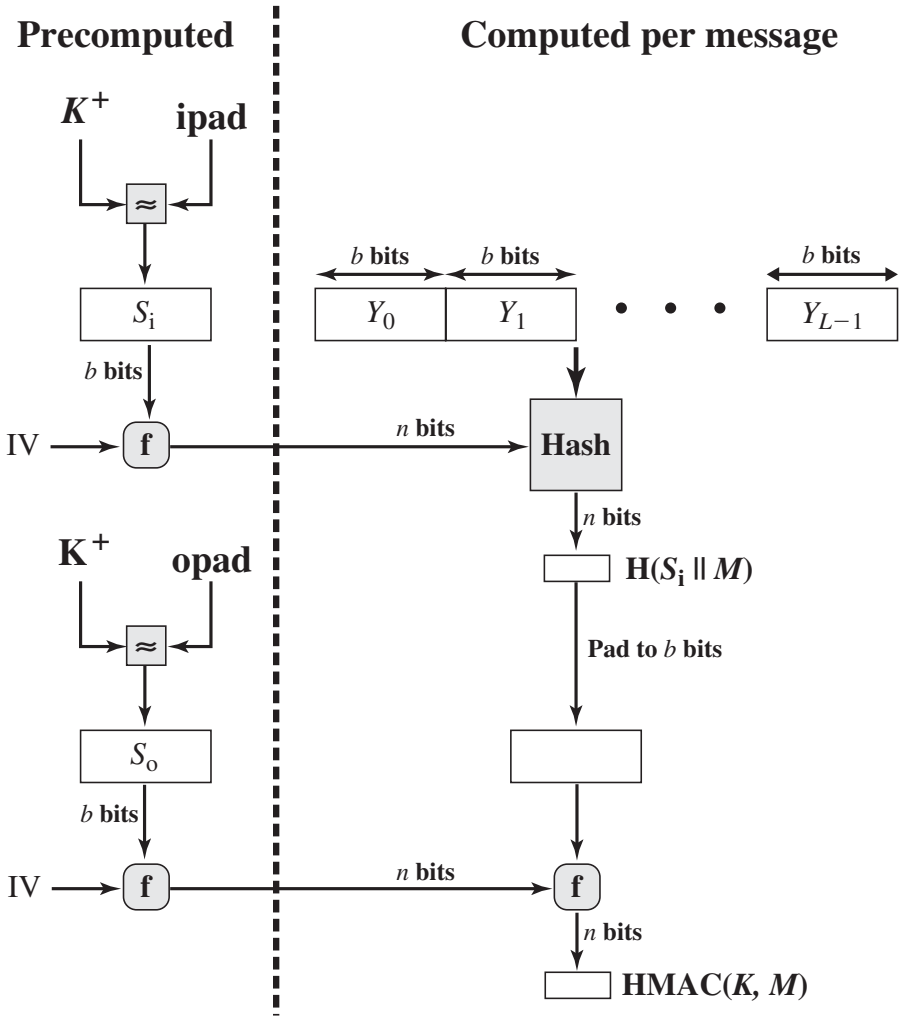


Figure 3.14 Efficient Implementation of HMAC

adversary gets $T_0 = \text{CBC}(K, \mathbf{0}) \oplus K_1$; $T_1 = \text{CBC}(K, \mathbf{1}) \oplus K_1$ and $T_2 = \text{CBC}(K, [\text{CBC}(K, \mathbf{1})]) \oplus K_1$. Show that the adversary can compute the correct MAC for the (unqueried) message $\mathbf{0} \| (T_0 \oplus T_1)$.

3.13 Prior to the discovery of any specific public-key schemes, such as RSA, an existence proof was developed whose purpose was to demonstrate that public-key encryption is possible in theory. Consider the functions $f_1(x_1) = z_1$; $f_2(x_2, y_2) = z_2$; $f_3(x_3, y_3) = z_3$, where all values are integers with $1 \le x_i, y_i, z_i \le N$. Function f_1 can be represented by a vector M1 of length N in which the kth entry is the value of $f_1(k)$. Similarly, f_2 and f_3 can be represented by $N \times N$ matrices M2 and M3. The intent is to represent the encryption/decryption process by table lookups for tables with very large values of N. Such tables would be impractically huge but in principle could be constructed. The scheme works as follows: construct M1 with a random permutation of all integers between 1 and N; that is, each integer appears exactly once in M1. Construct M2 so that each row contains a random permutation of the first N integers. Finally, fill in M3 to satisfy the condition:

$$f_3(f_2(f_1(k), p), k) = p \quad \text{for all } k, p \text{ with } 1 \le k, p \le N$$

In words,

1. M1 takes an input k and produces an output x.
2. M2 takes inputs x and p giving output z.
3. M3 takes inputs z and k and produces p.

The three tables, once constructed, are made public.

a. It should be clear that it is possible to construct M3 to satisfy the preceding condition. As an example, fill in M3 for the following simple case:

M1 =

5
4
2
3
1

M2 =

5	2	3	4	1
4	2	5	1	3
1	3	2	4	5
3	1	4	2	5
2	5	3	4	1

M3 =

Convention: The ith element of M1 corresponds to $k = i$. The ith row of M2 corresponds to $x = i$; the jth column of M2 corresponds to $p = j$. The ith row of M3 corresponds to $z = i$; the jth column of M3 corresponds to $k = j$.

b. Describe the use of this set of tables to perform encryption and decryption between two users.

c. Argue that this is a secure scheme.

3.14 Perform encryption and decryption using the RSA algorithm (Figure 3.10) for the following:

a. $p = 3; q = 11, e = 7; M = 5$
b. $p = 5; q = 11, e = 3; M = 9$
c. $p = 7; q = 11, e = 17; M = 8$
d. $p = 11; q = 13, e = 11; M = 7$
e. $p = 17; q = 31, e = 7; M = 2$

Hint: Decryption is not as hard as you think; use some finesse.

3.15 In a public-key system using RSA, you intercept the ciphertext $C = 10$ sent to a user whose public key is $e = 5, n = 35$. What is the plaintext M?

3.16 In an RSA system, the public key of a given user is $e = 31, n = 3599$. What is the private key of this user?

3.17 Suppose we have a set of blocks encoded with the RSA algorithm and we don't have the private key. Assume $n = pq$, e is the public key. Suppose also someone tells us they know one of the plaintext blocks has a common factor with n. Does this help us in any way?

3.18 Show how RSA can be represented by matrices M1, M2, and M3 of Problem 3.4.

3.19 Consider the following scheme.

1. Pick an odd number, E.
2. Pick two prime numbers, P and Q, where $(P - 1)(Q - 1) - 1$ is evenly divisible by E.
3. Multiply P and Q to get N.
4. Calculate $D = \dfrac{(P - 1)(Q - 1)(E - 1) + 1}{E}$.

Is this scheme equivalent to RSA? Show why or why not.

3.20 Suppose Bob uses the RSA cryptosystem with a very large modulus n for which the factorization cannot be found in a reasonable amount of time. Suppose Alice sends a message to Bob by representing each alphabetic character as an integer between 0 and 25 (A $\rightarrow$ 0, . . ., Z $\rightarrow$ 25), and then encrypting each number separately using RSA with large e and large n. Is this method secure? If not, describe the most efficient attack against this encryption method.

3.21 Consider a Diffie-Hellman scheme with a common prime $q = 11$ and a primitive root $\alpha = 2$.

a. If user A has public key $Y_A = 9$, what is A's private key X_A?
b. If user B has public key $Y_B = 3$, what is the shared secret key K?