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Parallel Programming Models and Dependences

Concurrency and Parallelism — 2019-20
Master in Computer Science
(Mestrado Integrado em Eng. Informática)

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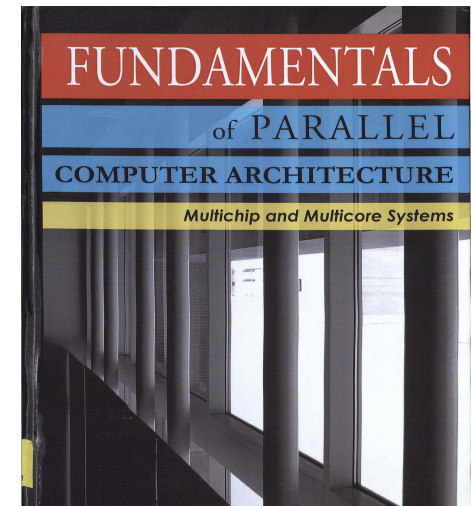
Source: Parallel Computing, CIS 410/510, Department of Computer and Information Science

Outline

- Parallel programming models
- Statement dependences
- Loop dependences

– Bibliography:

- **(Part of) Chapter 4** of book
Yan Solihin;
Fundamentals of Parallel
Computer Architecture;
Solihin Books (2009);
ISBN: 978-0-98-416300-7



Parallelism, Correctness, and Dependences

- Parallel execution shall always be constrained by the sequence of operations needed to be performed for a correct result
- Parallel execution must address control, data, and system dependences
- A *dependence* arises when one operation depends on an earlier operation to complete and produce a result before this later operation can be performed
 - We extend this notion of dependence to resources since some operations may depend on certain resources (e.g., due to where data is located)

Executing Two Statements in Parallel

- Want to execute two statements in parallel
- On one processor:

Processor 1:
Statement 1;
Statement 2;

- On two processors:

Processor 1:
Statement 1;

Processor 2:
Statement 2;

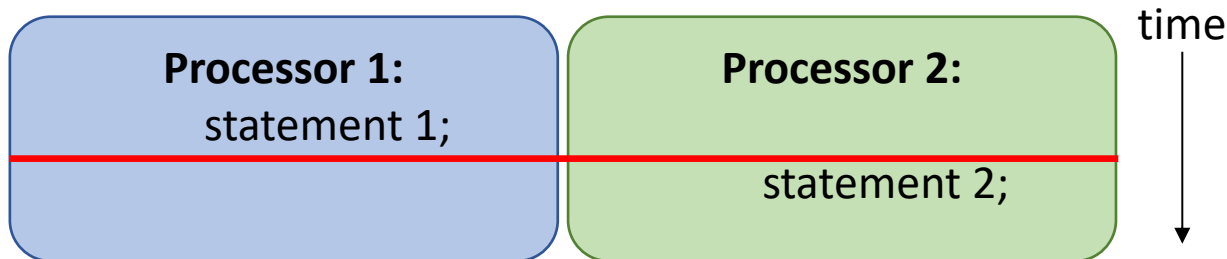
- Fundamental (*concurrent*) execution assumption
 - Processors execute independent of each other
 - No assumptions made about speed of processor execution

Sequential Consistency in Parallel Execution

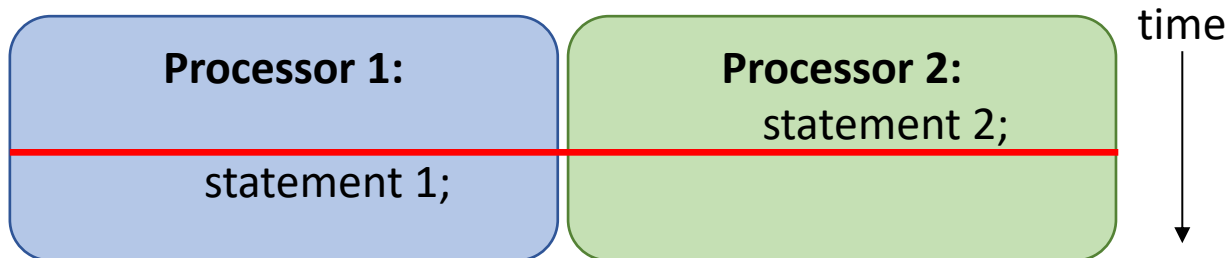
- Parallel execution of



- Case 1:

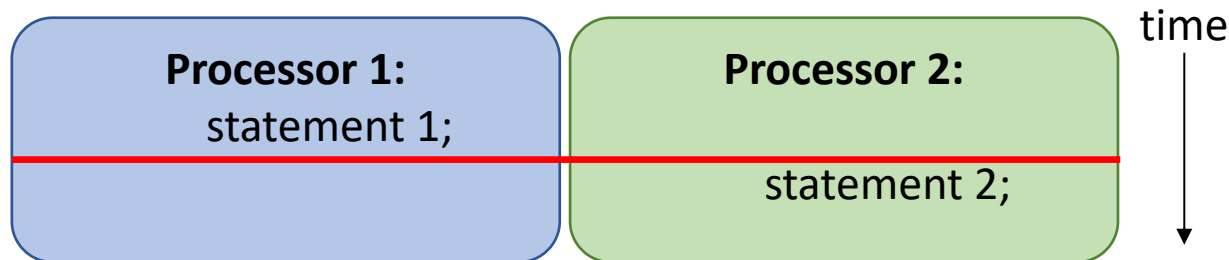


- Case 2:

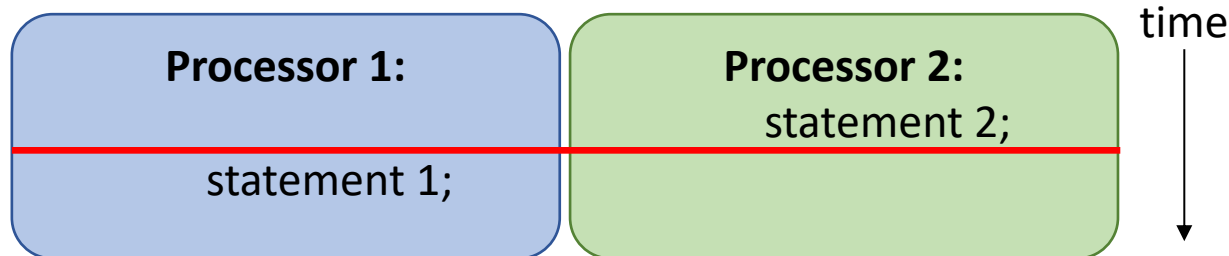


Sequential Consistency in Parallel Execution

- Sequential consistency
 - Statements execution does not interfere with each other
 - Computation result equal to either “Case 1” or “Case 2”
- Case 1:



- Case 2:



Independent versus Dependent

- When the execution of

```
statement1;  
statement2;
```

is equivalent to

```
statement2;  
statement1;
```

- Their order of execution must not matter!
- That means the statements are *independent* of each other
- Two statements are *dependent* when the order of their execution affects the computation outcome

True Dependence and Anti-Dependence

- Given statements S1 and S2 written as,
S1;
S2;

- S2 has a *true (flow) dependence* on S1
if and only if **S2 reads a value written by S1**
(RAW – Read After Write)

$$\begin{array}{c} x = \\ \vdots \\ = x \end{array} \quad \left. \begin{array}{c} \text{---} \\ \uparrow \\ \leftarrow \end{array} \right\} \delta$$

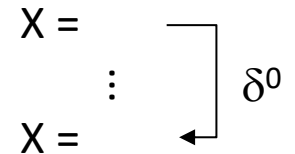
- S2 has a *anti-dependence* on S1
if and only if **S2 writes a value read by S1**
(WAR – Write After Read)

$$\begin{array}{c} = x \\ \vdots \\ x = \end{array} \quad \left. \begin{array}{c} \text{---} \\ \uparrow \\ \leftarrow \end{array} \right\} \delta^{-1}$$

Output Dependence

- Given statements S1 and S2 written as,
S1;
S2;

- S2 has an *output dependence* on S1
if and only if **S2 writes a variable written by S1**
(WAW – Write After Write)



- Anti- and output dependences are “name” dependences
 - How can we get rid of anti- and output dependences?

Examples

- Example 1

S1: a=1;
S2: b=1;

- Statements are independent

- Example 2

S1: a=1;
S2: b=a;

- Dependent (*true (flow) dependence*)
 - Second is dependent on first
 - Can you remove dependence?

- Example 3

S1: a=f(x);
S2: a=b;

- Dependent (*output dependence*)
 - Second is dependent on first
 - Can you remove dependence? How?

- Example 4

S1: a=b;
S2: b=1;

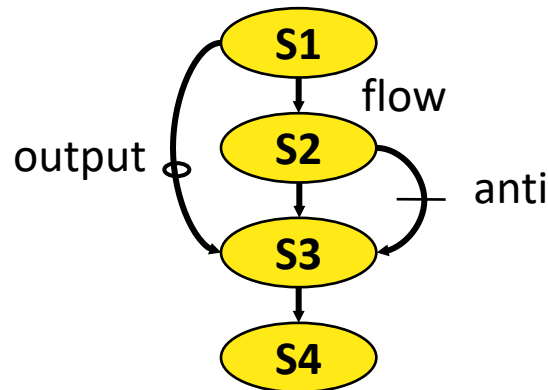
- Dependent (*anti-dependence*)
 - First is dependent on second
 - Can you remove dependence? How?

Statement Dependence Graphs

- Can use graphs to show dependence relationships

- Example

S1: a=1;
S2: b=a;
S3: a=b+1;
S4: c=a;



- $S_1 \delta S_2$: S_2 is flow-dependent on S_1
- $S_1 \delta^0 S_3$: S_3 is output-dependent on S_1
- $S_2 \delta^{-1} S_3$: S_3 is anti-dependent on S_2

When can two statements execute in parallel?

- Statements S1 and S2 can execute in parallel if and only if there are *no dependences* between them, i.e., no
 - *True dependences*; nor
 - *Anti-dependences*; nor
 - *Output dependences*.
- Some dependences can be removed by modifying the program
 - Rearranging statements
 - Eliminating statements

How do you compute dependences?

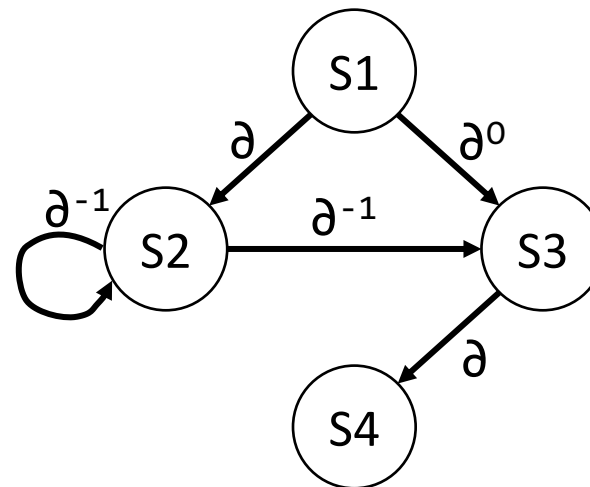
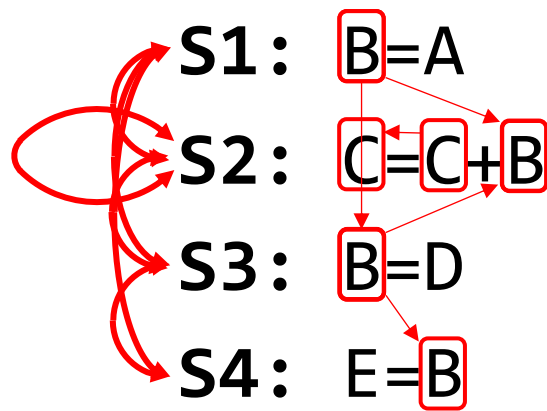
- Data dependence relations can be found by comparing the IN and OUT sets of each node
- The IN and OUT sets of a statement S are defined as:
 - $IN(S)$: set of memory locations (variables) that may be used (read) in S
 - $OUT(S)$: set of memory locations (variables) that may be modified (written) by S
- Note that these sets include all memory locations that may be fetched or modified
 - As such, the sets can be conservatively large

IN / OUT Sets and Computing Dependences

- Assuming that there is an execution path from **S1** to **S2**, the following shows how to intersect their IN and OUT sets to test for data dependence

$out(S_1) \cap in(S_2) \neq \emptyset$	$S_1 \delta S_2$	flow dependence
$in(S_1) \cap out(S_2) \neq \emptyset$	$S_1 \delta^{-1} S_2$	anti - dependence
$out(S_1) \cap out(S_2) \neq \emptyset$	$S_1 \delta^0 S_2$	output dependence

Example




Loop-Level Parallelism

- Significant parallelism can be identified **within loops**

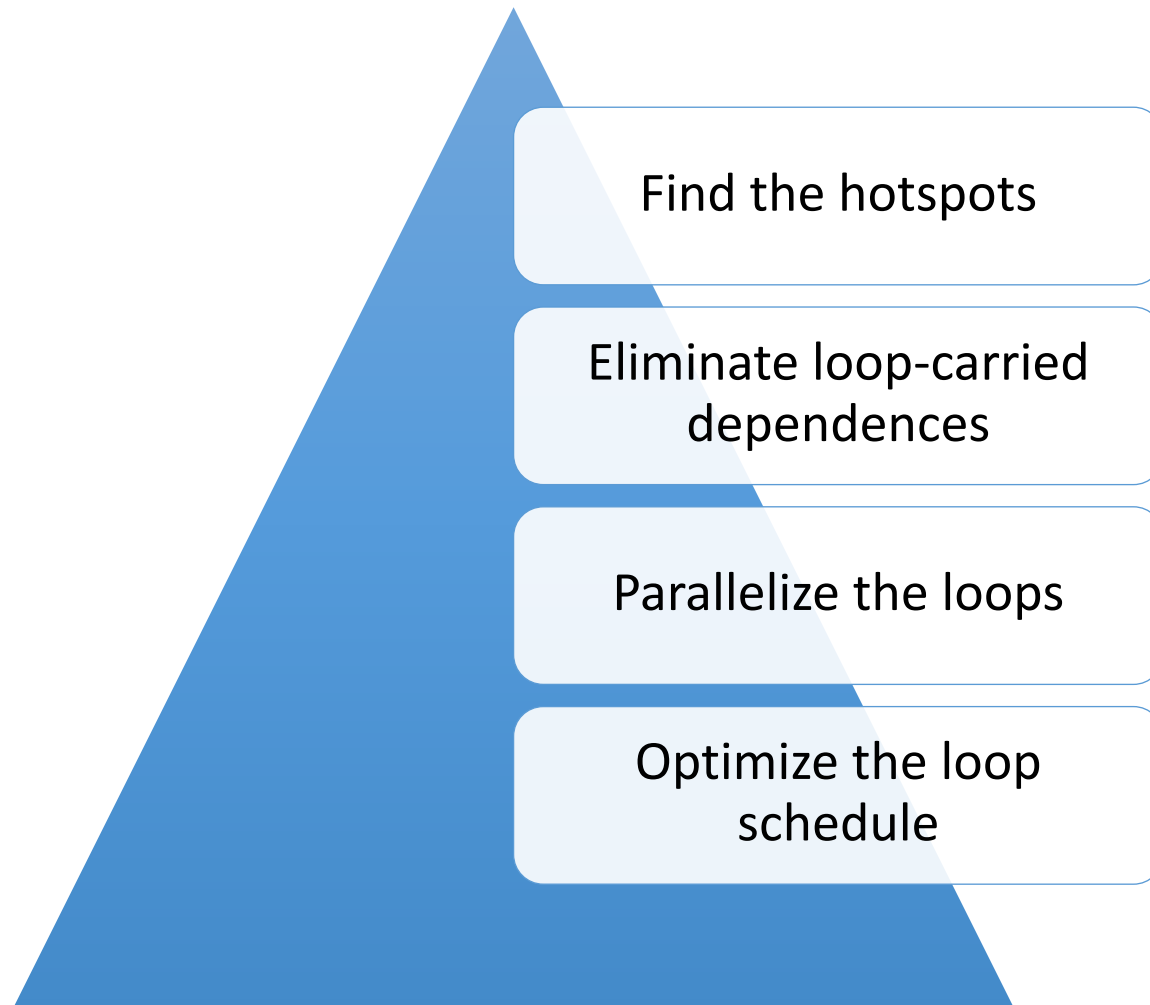
```
for (i=0; i<100; i++)  
    S1: a[i] = i;
```

```
parallel_for (i=0; i<100; i++)  
{  
    S1: a[i] = i;  
    S2: b[i] = 2*a[i];  
}
```



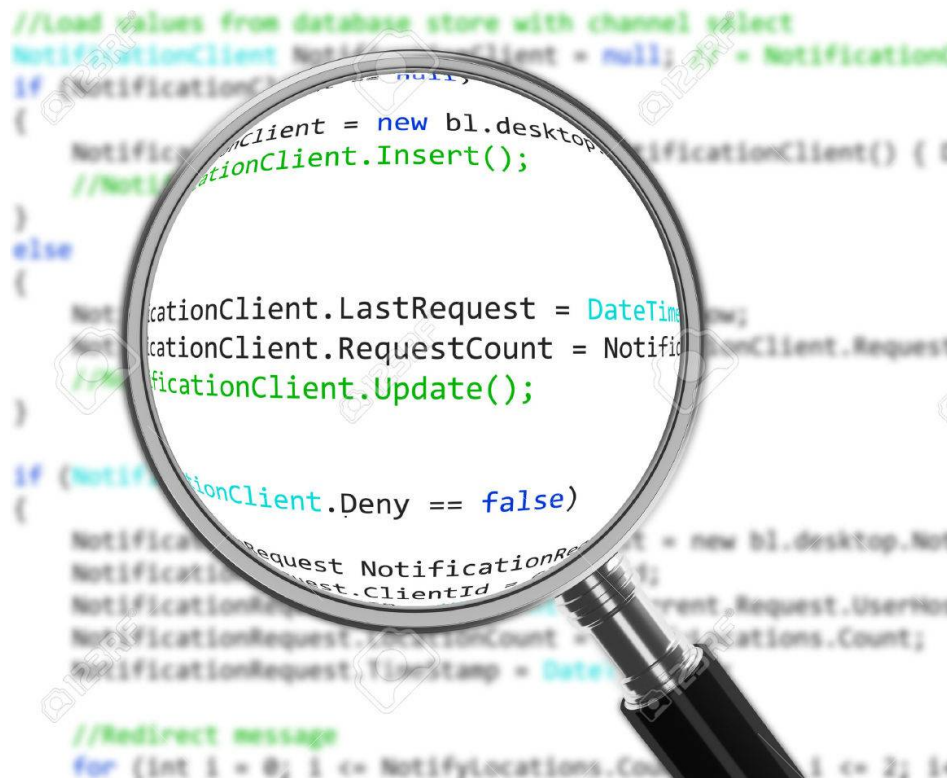
- Dependences? What about i , the loop index?
- DOALL* loop (a.k.a. *foreach* loop)
 - All iterations are independent of each other
 - All statements will be executed in parallel at the same time
 - Is this really true?

General Approach for Loop Parallelism

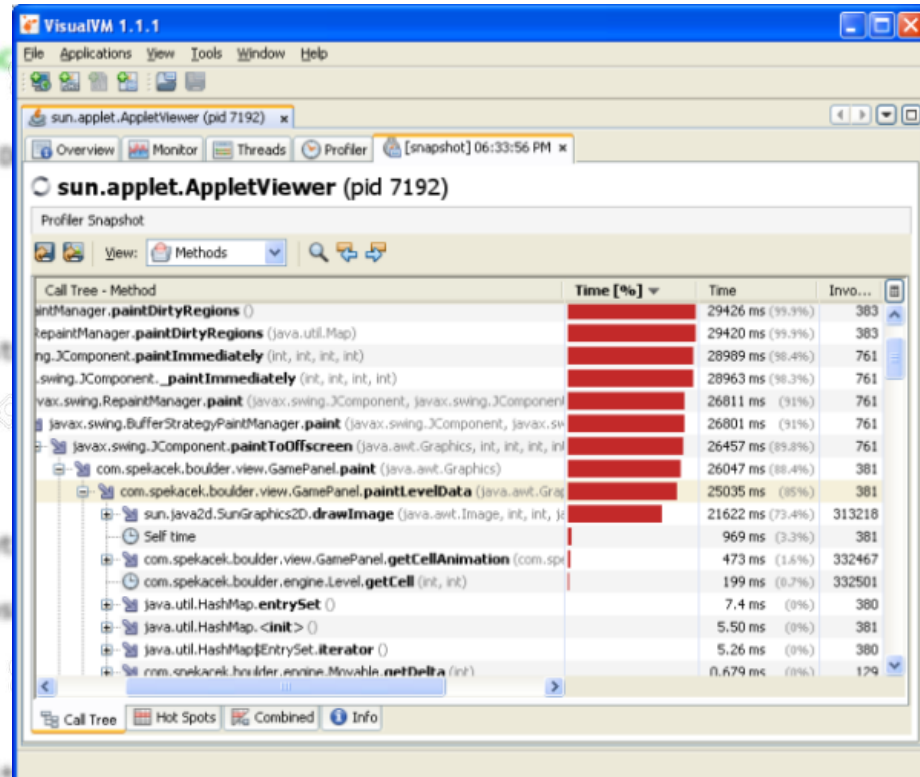


Find the hotspots

- By code inspection
- By using performance analysis tools



```
//Load values from database store with channel select
NotificationClient NotificationClient = null;
if (NotificationClient == null)
{
    NotificationClient = new bl.desktop.NotificationClient();
    NotificationClient.Insert();
    //NotificationClient.Insert();
}
else
{
    NotificationClient.LastRequest = DateTime.Now;
    NotificationClient.RequestCount = NotificationClient.RequestCount + 1;
    NotificationClient.Update();
}
if (NotificationClient.Deny == false)
{
    NotificationClient.LastRequest = DateTime.Now;
    NotificationClient.RequestCount = NotificationClient.RequestCount + 1;
    NotificationClient.Update();
}
//Redirect message
for (int i = 0; i <= NotificationLocations.Count; i++)
```



Eliminate loop-carried dependences

- Statements dependences include: true dependences, anti-dependences and output dependences.
- Loop dependences also include those, carried from one execution of the loop to another.

Loop Dependences

- A *loop-carried* dependence is a dependence between two statements instances in two different iterations of a loop
- Otherwise, it is *loop-independent*
- Loop-carried dependences can prevent loop iteration parallelization

Loop Dependences

- A *loop-carried* dependence is a dependence between two statements instances in two different iterations of a loop

```
S1: a = 5;  
S2: b = a;
```

True dependence — the memory location 'a' is written (in S1) before it is read (in S2)

S1 δ S2

```
for (i=1; i<n; i++) {  
    S1: a[i] = a[i-1];  
}
```

True dependence — a memory location 'a[j]' is written before it is read in the next iteration of the loop

S1[j] δ S1[j+1]

Loop Dependences

- A *loop-carried* dependence is a dependence between two statements instances in two different iterations of a loop

```
S1: b = a;  
S2: a = 5;
```

Anti-dependence — the memory location 'a' is read (in S1) before it is written (in S2)

$S1 \delta^{-1} S2$

```
for (i=0; i<n-1; i++) {  
    S1: a[i] = a[i+1];  
}
```

Anti-dependence — a memory location 'a[j]' is read before it is written in the next iteration of the loop

$S1[j] \delta^{-1} S1[j+1]$

Loop Dependences

- A *loop-carried* dependence is a dependence between two statements instances in two different iterations of a loop

```
S1: c = 8;  
S2: c = 15;
```

Output dependence — the same memory location 'c' is written (in S1) and then written once again (in S2)

S1 ∂^o S2

```
for (i=0; i<n; i++) {  
    S1: c[i] = i;  
    S2: c[i+1] = 5;  
}
```


Output dependence — the same memory location 'a[j]' is written (in S2) and then written again in the next iteration of the loop (in S1)

S2[j] ∂^o S1[j+1]

Loop dependences: examples

- The following loop cannot be parallelized (without rewriting)

```
a[0] = 1;  
for (i=1; i<N; i++) {  
    a[i] = a[i] + a[i-1];  
}
```



i=1: $a[1] = a[1] + a[0];$

i=2: $a[2] = a[2] + a[1];$

i=3: $a[3] = a[3] + a[2];$

...

Each iteration depends on the result of the preceding iteration

Detecting dependences

- Analyze how each variable is used within a loop iteration:
- Is the variable read and never written?
=> no dependences!
- For each written variable: can there be any accesses in other iterations than the current?
=> there are dependences!

Simple rule of thumb

- A loop that matches the following criteria has no dependences and can be parallelized:
 1. All assignments to shared data are to arrays:
 2. Each element is assigned by at most one iteration; and
 3. No iteration reads elements assigned by any other iteration.

Example 1

- Is this loop parallelizable?

```
for (i=1; i<N; i+=2) {  
    a[i] = a[i] + a[i-1];  
}
```

1. All assignments to shared data are to arrays:
2. Each element is assigned by at most one iteration; and
3. No iteration reads elements assigned by any other iteration.

i=1: $a[1] = a[1] + a[0];$

i=3: $a[3] = a[3] + a[2];$

i=5: $a[5] = a[5] + a[4];$

...

No dependences!
YES!! It is parallelizable!

Example 2

- Is this loop parallelizable?

```
for (i=0; i<N/2; i++) {  
    a[i] = a[i] + a[i+N/2];  
}
```

1. All assignments to shared data are to arrays:
2. Each element is assigned by at most one iteration; and
3. No iteration reads elements assigned by any other iteration.

i=0: $a[0] = a[0] + a[0+N/2];$

i=1: $a[1] = a[1] + a[1+N/2];$

...

i=N/2-1: $a[N/2-1] = a[N/2-1] + a[N-1];$

No dependences!

YES!! It is parallelizable!

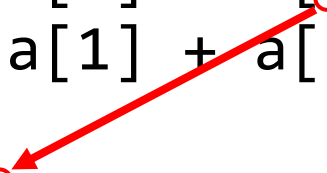
Example 3

- Is this loop parallelizable?

```
for (i=0; i<=N/2; i++) {  
    a[i] = a[i] + a[i+N/2];  
}
```

1. All assignments to shared data are to arrays:
2. Each element is assigned by at most one iteration; and
3. No iteration reads elements assigned by any other iteration.

$i=0$: $a[0] = a[0] + a[0+N/2]$;
 $i=1$: $a[1] = a[1] + a[1+N/2]$;
...
 $i=N/2$: $a[N/2] = a[N/2] + a[N]$;



Loop carried true
dependence
It is NOT parallelizable!

Example 4

- Is this loop parallelizable?

```
for (i=0; i<N; i++) {  
    a[idx[i]] = a[idx[i]] + b[idx[i]];  
}
```

i=0: $a[?_1] = a[?_1] + b[?_1];$

i=1: $a[?_2] = a[?_2] + b[?_2];$

i=3: $a[?_3] = a[?_3] + b[?_3];$

...

Don't know which index is
accessed in each iteration
of the loop.

It is NOT parallelizable!

Removing dependences 1

- How to remove this dependence?

```
for (i=0; i<=N/2; i++) {  
    a[i] = a[i] + a[i+N/2];  
}
```

Take the
dependent
iteration out
of the loop

```
for (i=0; i<N/2; i++) {  
    a[i] = a[i] + a[i+N/2];  
}  
a[N/2] = a[N/2] + a[N];
```

Removing dependences 2

- How to remove this dependence?

```
for (i=0; i<N; i++) {  
    x = (b[i] + c[i]) / 2;  
    a[i] = a[i+1] + x;  
}
```

```
for (i=0; i<N; i++) {  
    x = (b[i] + c[i]) / 2;  
    a[i] = a[i+1] + x;  
}
```

True dependence inside the loop (x)

Output dependence between iterations (x)

Anti-dependence between iterations (x)

Anti-dependence between iterations (a[i])

- To remove the dependences on 'x' privatize it

Removing dependences 2

- How to remove this dependence?

```
for (i=0; i<N; i++) {  
    x = (b[i] + c[i]) / 2;  
    a[i] = a[i+1] + x;  
}
```

```
for (i=0; i<N; i++) {  
    int x = (b[i] + c[i]) / 2;  
    a[i] = a[i+1] + x;  
}
```

Anti-dependence between iterations (a[i])

- To remove the dependence on 'a[i]'
make copy of 'a'

Removing dependences 2

- How to remove this dependence?

```
for (i=0; i<N; i++) {  
    x = (b[i] + c[i]) / 2;  
    a[i] = a[i+1] + x;  
}
```

```
for (i=0; i<N; i++) {  
    a2[i] = a[i+1];  
}
```

```
for (i=0; i<N; i++) {  
    int x = (b[i] + c[i]) / 2;  
    a[i] = a2[i] + x;  
}
```

Anti-dependence between iterations (a[i])

- Both 'for' are parallelizable!! *Should we do it?*

Removing dependences 3

- How to remove this dependence?

```
for (i=1; i<N; i++) {  
    b[i] += a[i-1];  
    a[i] += c[i];  
}
```

i=1: $b[1]=b[1]+a[0];$ $a[1]=a[1]+c[1]$

i=2: $b[2]=b[2]+a[1];$ $a[2]=a[2]+c[2]$

...

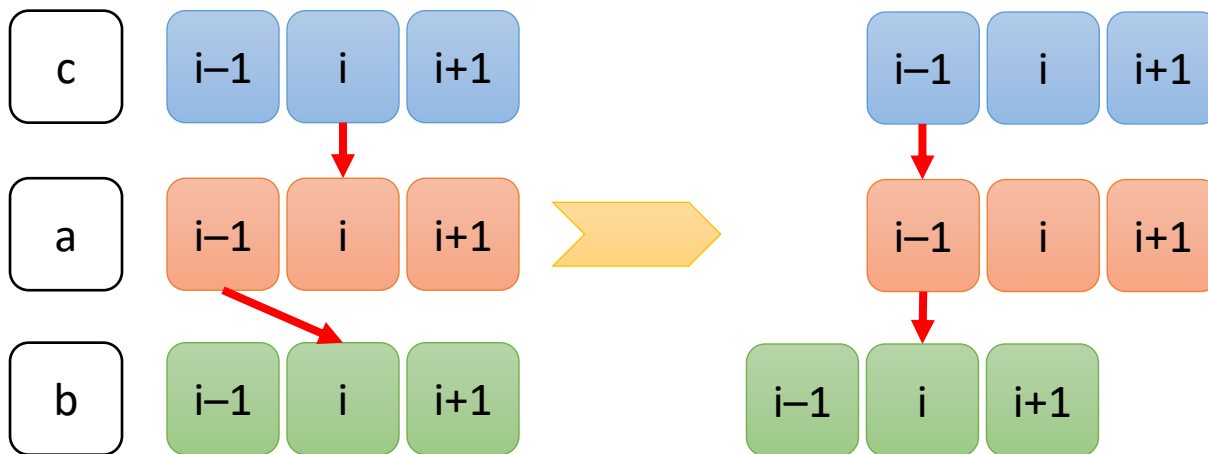
i=N-1 : $b[N-1]=b[N-1]+a[N-2];$ $a[N-1]=a[N-1]+c[N-1]$

Removing dependences 3

- How to remove this dependence?

```
for (i=1; i<N; i++) {  
    b[i] += a[i-1];  
    a[i] += c[i];  
}
```

Use *software pipelining*!



Removing dependences 3

- How to remove this dependence?

```
for (i=1; i<N; i++) {  
    b[i] += a[i-1];  
    a[i] += c[i];  
}
```

```
b[1] += a[0];  
for (i=1; i<N-1; i++) {  
    a[i] += c[i];  
    b[i+1] += a[i];  
}  
a[N] += c[N];
```

Removing dependences 4



**Not all loops
can be made parallel!**

The END
