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Introduction to Computer Graphics with WebGL

- Ed Angel
- Professor Emeritus of Computer Science
- Founding Director, Arts, Research, Technology and Science Laboratory
- University of New Mexico



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Representation

- Ed Angel
- Professor Emeritus of Computer Science,
- University of New Mexico



Objectives

- Introduce concepts such as dimension and basis
- Introduce coordinate systems for representing vectors spaces and frames for representing affine spaces
- Discuss change of frames and bases



Linear Independence

- A set of vectors $v_1, v_2, \dots, v_n$ is *linearly independent* if

$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n = 0 \text{ iff } \alpha_1 = \alpha_2 = \dots = 0$$

- If a set of vectors is linearly independent, we cannot represent one in terms of the others
- If a set of vectors is linearly dependent, at least one can be written in terms of the others



Dimension

- In a vector space, the maximum number of linearly independent vectors is fixed and is called the *dimension* of the space
- In an n -dimensional space, any set of n linearly independent vectors form a *basis* for the space
- Given a basis $v_1, v_2, \dots, v_n$, any vector v can be written as

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

where the $\{\alpha_i\}$ are unique



Representation

- Until now we have been able to work with geometric entities without using any frame of reference, such as a coordinate system
- Need a frame of reference to relate points and objects to our physical world.

For example, where is a point? Can't answer without a reference system

World coordinates

Camera coordinates



Coordinate Systems

- Consider a basis $v_1, v_2, \dots, v_n$
- A vector is written $v = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$
- The list of scalars $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is the *representation* of v with respect to the given basis
- We can write the representation as a row or column array of scalars

$$\mathbf{a} = [\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_n]^T = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$$



Example

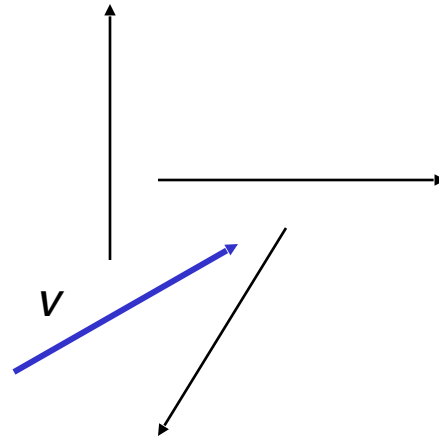
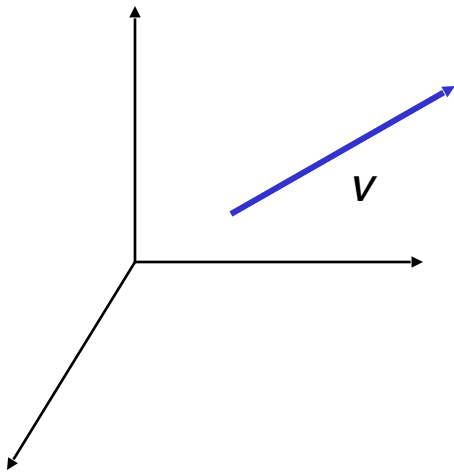
- $\mathbf{v} = 2\mathbf{v}_1 + 3\mathbf{v}_2 - 4\mathbf{v}_3$
- $\mathbf{a} = [2 \ 3 \ -4]^T$
- Note that this representation is with respect to a particular basis
- For example, in WebGL we will start by representing vectors using the object basis but later the system needs a representation in terms of the camera or eye basis



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Coordinate Systems

- Which is correct?

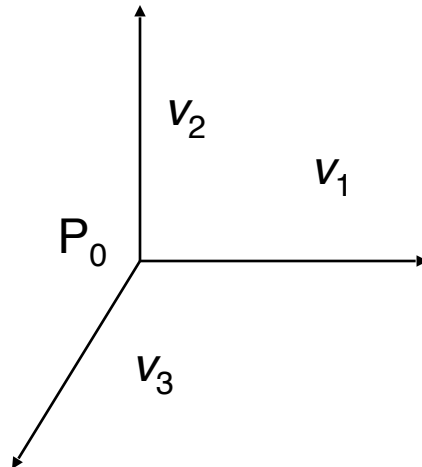


- Both are because vectors have no fixed location



Frames

- A coordinate system is insufficient to represent points
- If we work in an affine space we can add a single point, the *origin*, to the basis vectors to form a *frame*





Representation in a Frame

- Frame determined by (P_0, v_1, v_2, v_3)
- Within this frame, every vector can be written as

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

- Every point can be written as

$$P = P_0 + \beta_1 v_1 + \beta_2 v_2 + \dots + \beta_n v_n$$

Confusing Points and Vectors

Consider the point and the vector

$$P = P_0 + \beta_1 v_1 + \beta_2 v_2 + \dots + \beta_n v_n$$

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

They appear to have the similar representations

$$\mathbf{p} = [\beta_1 \ \beta_2 \ \beta_3] \quad \mathbf{v} = [\alpha_1 \ \alpha_2 \ \alpha_3]$$

which confuses the point with the vector

A vector has no position

Vector can be placed anywhere

