APPENDIX A
Real Numbers, Intervals, and Inequalities

EXERCISE SET A

1. **(a)** rational **(b)** integer, rational **(c)** integer, rational
(d) rational **(e)** integer, rational **(f)** irrational
(g) rational **(h)** 467 integer, rational

2. **(a)** irrational **(b)** rational **(c)** rational **(d)** rational

3. **(a)** $x = 0.123123123\ldots$, $1000x = 123 + x$, $x = 123/999 = 41/333$
(b) $x = 12.7777\ldots$, $10(x-12) = 7 + (x-12)$, $9x = 115$, $x = 115/9$
(c) $x = 38.07818181\ldots$, $100x = 3807.81818181\ldots$, $99x = 100x - x = 3769.74$,
$x = \dfrac{3769.74}{99} = \dfrac{376974}{9900} = \dfrac{20943}{550}$
(d) $\dfrac{4296}{10000} = \dfrac{537}{1250}$

4. $x = 0.99999\ldots$, $10x = 9 + x$, $9x = 9$, $x = 1$

5. **(a)** If r is the radius, then $D = 2r$ so $\left(\dfrac{8}{9}D\right)^2 = \left(\dfrac{16}{9}r\right)^2 = \dfrac{256}{81}r^2$. The area of a circle of radius r is πr^2 so $256/81$ was the approximation used for π.
(b) $22/7 \approx 3.1429$ is better than $256/81 \approx 3.1605$.

6. **(a)** $\dfrac{223}{71} < \dfrac{333}{106} < \dfrac{63}{25}\left(\dfrac{17+15\sqrt{5}}{7+15\sqrt{5}}\right) < \dfrac{355}{113} < \dfrac{22}{7}$

(b) Ramanujan's **(c)** Athoniszoon's **(d)** Ramanujan's

7.

Line	2	3	4	5	6	7
Blocks	3, 4	1, 2	3, 4	2, 4, 5	1, 2	3, 4

8.

Line	1	2	3	4	5
Blocks	all blocks	none	2, 4	2	2, 3

9. **(a)** always correct (add -3 to both sides of $a \le b$)
(b) not always correct (correct only if $a = b$)
(c) not always correct (correct only if $a = b$)
(d) always correct (multiply both sides of $a \le b$ by 6)
(e) not always correct (correct only if $a \ge 0$ or $a = b$)
(f) always correct (multiply both sides of $a \le b$ by the nonnegative quantity a^2)

10. **(a)** always correct
(b) not always correct (for example let $a = b = 0$, $c = 1$, $d = 2$)
(c) not always correct (for example let $a = 1$, $b = 2$, $c = d = 0$)

11. **(a)** all values because $a = a$ is always valid **(b)** none

12. $a = b$, because if $a \neq b$ then $a < b$ and $b < a$ are contradictory

13. **(a)** yes, because $a \le b$ means $a < b$ or $a = b$, thus $a < b$ certainly means $a \le b$
(b) no, because $a < b$ is false if $a = b$ is true

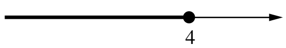
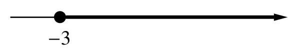
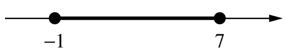
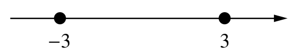
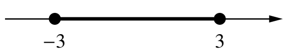
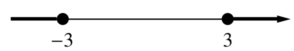
14. **(a)** $x^2 - 5x = 0$, $x(x-5) = 0$ so $x = 0$ or $x = 5$
(b) $-1, 0, 1, 2$ are the only integers that satisfy $-2 < x < 3$

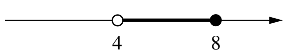
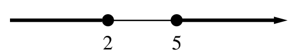
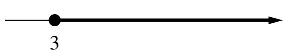
15. **(a)** $\{x : x \text{ is a positive odd integer}\}$ **(b)** $\{x : x \text{ is an even integer}\}$
(c) $\{x : x \text{ is irrational}\}$ **(d)** $\{x : x \text{ is an integer and } 7 \le x \le 10\}$

16. **(a)** not equal to A because 0 is not in A **(b)** equal to A
(c) equal to A because $(x-3)(x^2-3x+2) = 0$, $(x-3)(x-2)(x-1) = 0$ so $x = 1$, 2, or 3

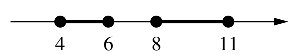
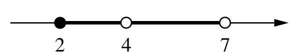
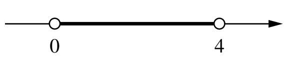
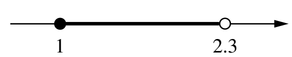
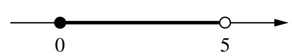
17. **(a)** false, there are points inside the triangle that are not inside the circle
(b) true, all points inside the triangle are also inside the square
(c) true **(d)** false **(e)** true
(f) true, a is inside the circle **(g)** true

18. **(a)** $\emptyset$, $\{a_1\}$, $\{a_2\}$, $\{a_3\}$, $\{a_1, a_2\}$, $\{a_1, a_3\}$, $\{a_2, a_3\}$, $\{a_1, a_2, a_3\}$ **(b)** $\emptyset$

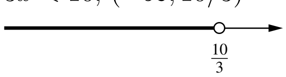
19. **(a)**  **(b)** 
(c)  **(d)** 
(e)  **(f)** 

20. **(a)**  **(b)** 
(b)  **(d)** none

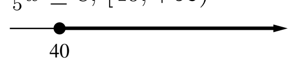
21. **(a)** $[-2, 2]$ **(b)** $(-\infty, -2) \cup (2, +\infty)$

22. **(a)**  **(b)** 
(c)  **(d)** 
(e)  **(f)** 
(g) **(h)** 

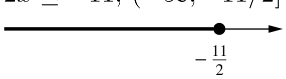
23. $3x < 10$; $(-\infty, 10/3)$



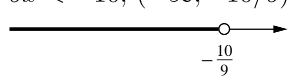
24. $\frac{1}{5}x \ge 8$; $[40, +\infty)$



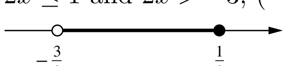
25. $2x \le -11$; $(-\infty, -11/2]$



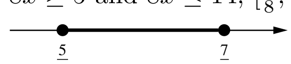
26. $9x < -10$; $(-\infty, -10/9)$



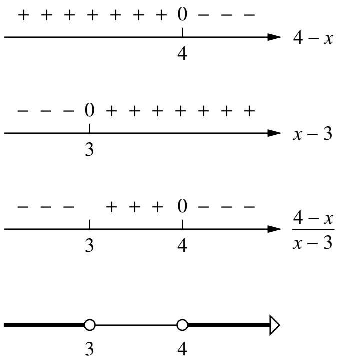
27. $2x \le 1$ and $2x > -3$; $(-3/2, 1/2]$



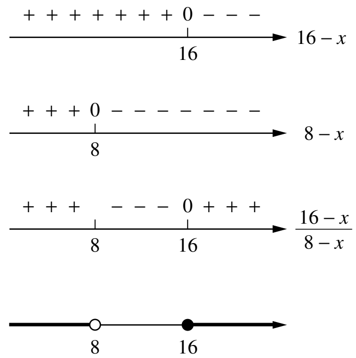
28. $8x \ge 5$ and $8x \le 14$; $[\frac{5}{8}, \frac{7}{4}]$



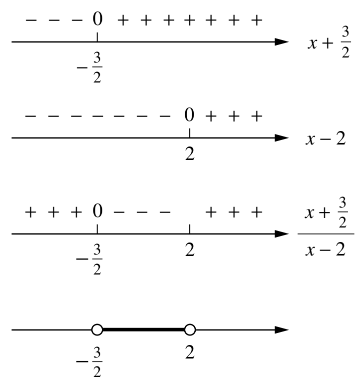
29. $\dfrac{x}{x-3} - 4 < 0,\ \dfrac{12-3x}{x-3} < 0,\ \dfrac{4-x}{x-3} < 0;$
$(-\infty, 3) \cup (4, +\infty)$



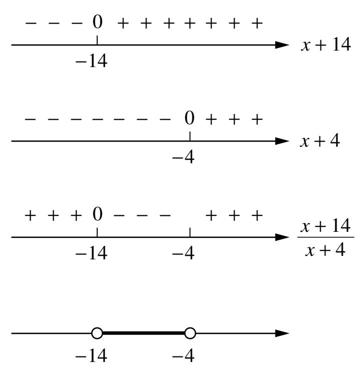
30. $\dfrac{x}{8-x} + 2 = \dfrac{16-x}{8-x} \geq 0;$
$(-\infty, 8) \cup [16, +\infty)$



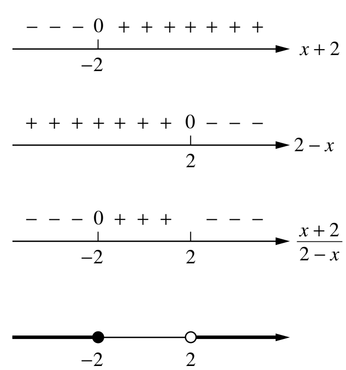
31. $\dfrac{3x+1}{x-2} - 1 = \dfrac{2x+3}{x-2} < 0,\ \dfrac{x+3/2}{x-2} < 0;$
$(-\frac{3}{2}, 2)$



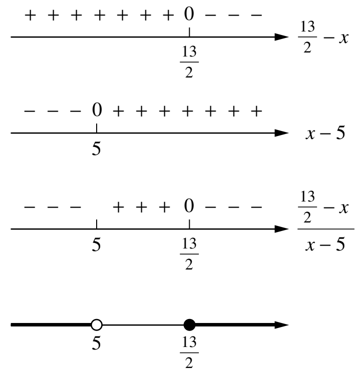
32. $\dfrac{x/2-3}{4+x} - 1 > 0,\ \dfrac{x-6}{4+x} - 2 > 0,\ \dfrac{x+14}{x+4} < 0;$
$(-14, -4)$



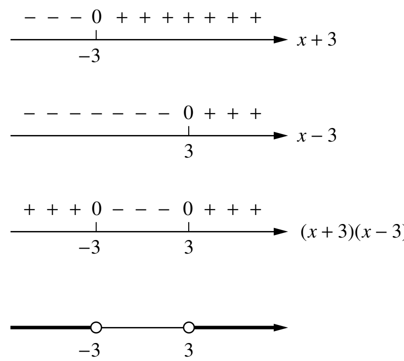
33. $\dfrac{4}{2-x} - 1 = \dfrac{x+2}{2-x} \leq 0; (-\infty, -2] \cup (2, +\infty)$



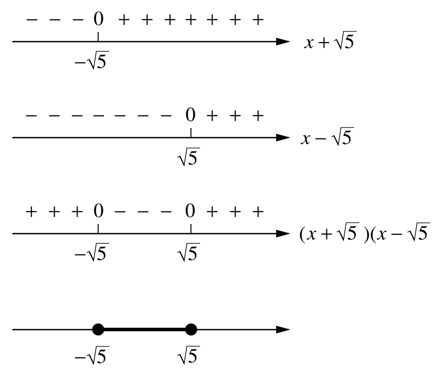
34. $\dfrac{3}{x-5} - 2 = \dfrac{13-2x}{x-5} \le 0,\ \dfrac{13/2 - x}{x-5} \le 0;$
$(-\infty, 5) \cup [\frac{13}{2}, +\infty)$



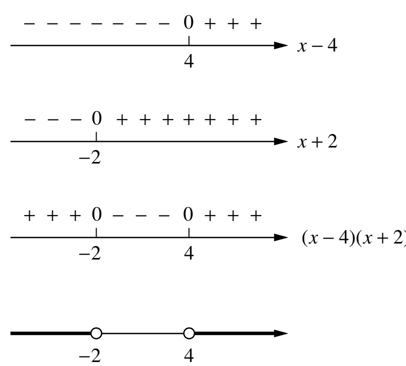
35. $x^2 - 9 = (x+3)(x-3) > 0;$
$(-\infty, -3) \cup (3, +\infty)$



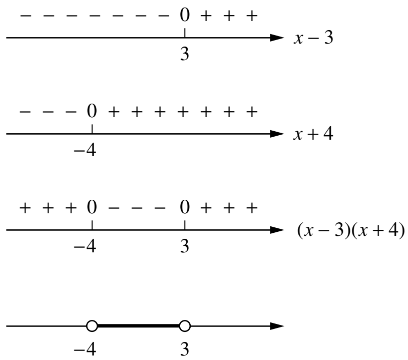
36. $x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5}) \le 0;\ [-\sqrt{5}, \sqrt{5}]$



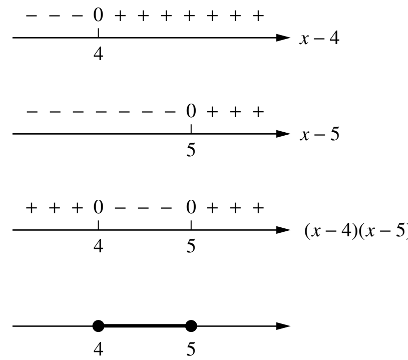
37. $(x-4)(x+2) > 0;\ (-\infty, -2) \cup (4, +\infty)$



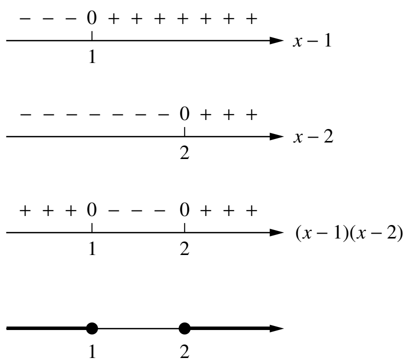
38. $(x-3)(x+4) < 0$; $(-4,3)$



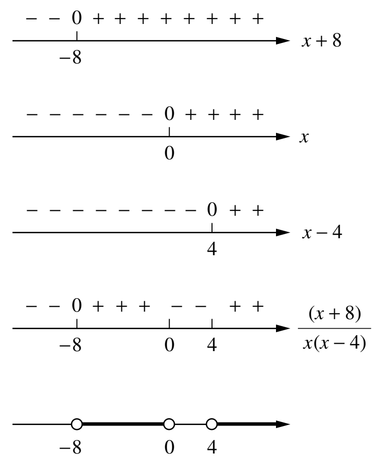
39. $(x-4)(x-5) \le 0$; $[4,5]$



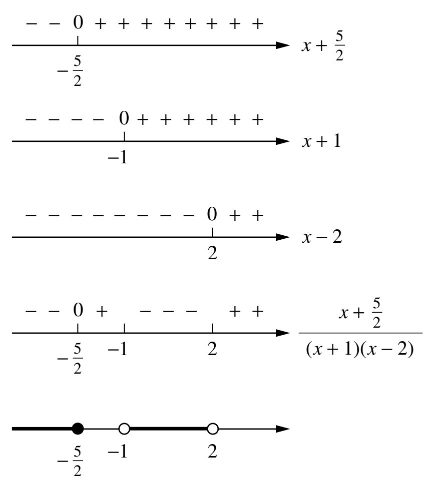
40. $(x-2)(x-1) \ge 0$;
$(-\infty,1] \cup [2,+\infty)$



41. $\dfrac{3}{x-4} - \dfrac{2}{x} = \dfrac{x+8}{x(x-4)} > 0$;
$(-8,0) \cup (4,+\infty)$



42. $\dfrac{1}{x+1} - \dfrac{3}{x-2} = \dfrac{-2x-5}{(x+1)(x-2)} \ge 0$,
$\dfrac{x+5/2}{(x+1)(x-2)} \le 0$;
$(-\infty,-\frac{5}{2}] \cup (-1,2)$



43. By trial-and-error we find that $x = 2$ is a root of the equation $x^3 - x^2 - x - 2 = 0$ so $x-2$ is a factor of $x^3 - x^2 - x - 2$. By long division we find that $x^2 + x + 1$ is another factor so $x^3 - x^2 - x - 2 = (x-2)(x^2+x+1)$. The linear factors of x^2+x+1 can be determined by first finding the roots of $x^2 + x + 1 = 0$ by the quadratic formula. These roots are complex numbers so $x^2+x+1 \neq 0$ for all real x; thus x^2+x+1 must be always positive or always negative. Since x^2+x+1 is positive when $x=0$, it follows that $x^2+x+1>0$ for all real x. Hence $x^3-x^2-x-2>0$, $(x-2)(x^2+x+1)>0$, $x-2>0$, $x>2$, so $S=(2,+\infty)$.

44. By trial-and-error we find that $x=1$ is a root of the equation $x^3-3x+2=0$ so $x-1$ is a factor of x^3-3x+2. By long division we find that x^2+x-2 is another factor so $x^3-3x+2=(x-1)(x^2+x-2)=(x-1)(x-1)(x+2)=(x-1)^2(x+2)$. Therefore we want to solve $(x-1)^2(x+2)\le 0$. Now if $x\neq 1$, then $(x-1)^2>0$ and so $x+2\le 0$, $x\le -2$. By inspection, $x=1$ is also a solution so $S=(-\infty,-2]\cup\{1\}$.

45. $\sqrt{x^2+x-6}$ is real if $x^2+x-6\ge 0$. Factor to get $(x+3)(x-2)\ge 0$ which has as its solution $x\le -3$ or $x\ge 2$.

46. $\dfrac{x+2}{x-1}\ge 0$; $(-\infty,-2]\cup(1,+\infty)$

47. $25\le \dfrac{5}{9}(F-32)\le 40$, $45\le F-32\le 72$, $77\le F\le 104$

48. **(a)** $n=2k$, $n^2=4k^2=2(2k^2)$ where $2k^2$ is an integer.
(b) $n=2k+1$, $n^2=4k^2+4k+1=2(2k^2+2k)+1$ where $2k^2+2k$ is an integer.

49. **(a)** Assume m and n are rational, then $m=\dfrac{p}{q}$ and $n=\dfrac{r}{s}$ where p, q, r, and s are integers so $m+n=\dfrac{p}{q}+\dfrac{r}{s}=\dfrac{ps+rq}{qs}$ which is rational because $ps+rq$ and qs are integers.

(b) (proof by contradiction) Assume m is rational and n is irrational, then $m=\dfrac{p}{q}$ where p and q are integers. Suppose that $m+n$ is rational, then $m+n=\dfrac{r}{s}$ where r and s are integers so $n=\dfrac{r}{s}-m=\dfrac{r}{s}-\dfrac{p}{q}=\dfrac{rq-ps}{sq}$. But $rq-ps$ and sq are integers, so n is rational which contradicts the assumption that n is irrational.

50. **(a)** Assume m and n are rational, then $m=\dfrac{p}{q}$ and $n=\dfrac{r}{s}$ where p, q, r, and s are integers so $mn=\dfrac{p}{q}\cdot\dfrac{r}{s}=\dfrac{pr}{qs}$ which is rational because pr and qs are integers.

(b) (proof by contradiction) Assume m is rational and nonzero and that n is irrational, then $m=\dfrac{p}{q}$ where p and q are integers and $p\neq 0$. Suppose that mn is rational, then $mn=\dfrac{r}{s}$ where r and s are integers so $n=\dfrac{r/s}{m}=\dfrac{r/s}{p/q}=\dfrac{rq}{ps}$. But rq and ps are integers, so n is rational which contradicts the assumption that n is irrational.

51. $a=\sqrt{2}, b=\sqrt{3}, c=\sqrt{6}, d=-\sqrt{2}$ are irrational, and $a+d=0$, a rational; $a+a=2\sqrt{2}$, an irrational; $ad=-2$, a rational; and $ab=c$, an irrational.

52. **(a)** irrational (Exercise 49(b)) **(b)** irrational (Exercise 50(b))
(c) rational by inspection; Exercise 51 gives no information
(d) $\sqrt{\pi}$ must be irrational, for if it were rational, then so would be $\pi=(\sqrt{\pi})^2$ by Exercise 50(a); but π is known to be irrational.

53. The average of a and b is $\frac{1}{2}(a+b)$; if a and b are rational then so is the average, by Exercise 49(a) and Exercise 50(a). On the other hand if $a=b=\sqrt{2}$ then the average of a and b is irrational, but the average of a and $-b$ is rational.

54. If $10^x = 3$, then $x > 0$ because $10^x \leq 1$ for $x \leq 0$. If $10^{p/q} = 3$ with p, q integers, then $10^p = 3^q$. Following Exercise 48, if $n = 2k$ is even, then $n^2, n^3, n^4, \ldots$ are even; and if $n = 2k + 1$ then $n^2, n^3, n^4, \ldots$ are odd. Since $10^p = 3^q$, the left side is even and the right side is odd, a contradiction.

55. $8x^3 - 4x^2 - 2x + 1$ can be factored by grouping terms:
$(8x^3 - 4x^2) - (2x - 1) = 4x^2(2x - 1) - (2x - 1) = (2x - 1)(4x^2 - 1) = (2x - 1)^2(2x + 1)$. The problem, then, is to solve $(2x - 1)^2(2x + 1) < 0$. By inspection, $x = 1/2$ is not a solution. If $x \neq 1/2$, then $(2x - 1)^2 > 0$ and it follows that $2x + 1 < 0$, $2x < -1$, $x < -1/2$, so $S = (-\infty, -1/2)$.

56. Rewrite the inequality as $12x^3 - 20x^2 + 11x - 2 \geq 0$. If a polynomial in x with integer coefficients has a rational zero $\dfrac{p}{q}$, a fraction in lowest terms, then p must be a factor of the constant term and q must be a factor of the coefficient of the highest power of x. By trial-and-error we find that $x = 1/2$ is a zero, thus $(x - 1/2)$ is a factor so

$$\begin{aligned}
12x^3 - 20x^2 + 11x - 2 &= (x - 1/2)\left(12x^2 - 14x + 4\right) \\
&= 2(x - 1/2)\left(6x^2 - 7x + 2\right) \\
&= 2(x - 1/2)(2x - 1)(3x - 2) = (2x - 1)^2(3x - 2).
\end{aligned}$$

Now to solve $(2x - 1)^2(3x - 2) \geq 0$ we first note that $x = 1/2$ is a solution. If $x \neq 1/2$ then $(2x - 1)^2 > 0$ and $3x - 2 \geq 0$, $3x \geq 2$, $x \geq 2/3$ so $S = [2/3, +\infty) \cup \{1/2\}$.

57. If $a < b$, then $ac < bc$ because c is positive; if $c < d$, then $bc < bd$ because b is positive, so $ac < bd$ (Theorem A.1(a)). (Note that the result is still true if one of a, b, c, d is allowed to be negative, that is $a < 0$ or $c < 0$.)

58. no, since the decimal representation is not repeating (the string of zeros does not have constant length)

APPENDIX B
Absolute Value

EXERCISE SET B

1. **(a)** 7 **(b)** $\sqrt{2}$ **(c)** k^2 **(d)** k^2

2. $\sqrt{(x-6)^2} = x - 6$ if $x \geq 6$, $\sqrt{(x-6)^2} = -(x-6) = -x + 6$ if $x < 6$

3. $|x-3| = |3-x| = 3 - x$ if $3 - x \geq 0$, which is true if $x \leq 3$

4. $|x+2| = x + 2$ if $x + 2 \geq 0$ so $x \geq -2$.

5. All real values of x because $x^2 + 9 > 0$.

6. $|x^2 + 5x| = x^2 + 5x$ if $x^2 + 5x \geq 0$ so $x(x+5) \geq 0$ which is true for $x \leq -5$ or $x \geq 0$.

7. $|3x^2 + 2x| = |x(3x+2)| = |x||3x+2|$. If $|x||3x+2| = x|3x+2|$, then $|x||3x+2| - x|3x+2| = 0$, $(|x| - x)|3x+2| = 0$, so either $|x| - x = 0$ or $|3x+2| = 0$. If $|x| - x = 0$, then $|x| = x$, which is true for $x \geq 0$. If $|3x+2| = 0$, then $x = -2/3$. The statement is true for $x \geq 0$ or $x = -2/3$.

8. $|6 - 2x| = |2(3-x)| = |2||3-x| = 2|x-3|$ for all real values of x.

9. $\sqrt{(x+5)^2} = |x+5| = x + 5$ if $x + 5 \geq 0$, which is true if $x \geq -5$.

10. $\sqrt{(3x-2)^2} = |3x-2| = |2-3x| = 2 - 3x$ if $2 - 3x \geq 0$ so $x \leq 2/3$.

13. **(a)** $|7-9| = |-2| = 2$ **(b)** $|3-2| = |1| = 1$
(c) $|6-(-8)| = |14| = 14$ **(d)** $|-3-\sqrt{2}| = |-(3+\sqrt{2})| = 3+\sqrt{2}$
(e) $|-4-(-11)| = |7| = 7$ **(f)** $|-5-0| = |-5| = 5$

14. $\sqrt{a^4} = \sqrt{(a^2)^2} = |a^2|$, but $|a^2| = a^2$ because $a^2 \geq 0$ so it is valid for all values of a.

15. **(a)** B is 6 units to the left of A; $b = a - 6 = -3 - 6 = -9$.
(b) B is 9 units to the right of A; $b = a + 9 = -2 + 9 = 7$.
(c) B is 7 units from A; either $b = a + 7 = 5 + 7 = 12$ or $b = a - 7 = 5 - 7 = -2$. Since it is given that $b > 0$, it follows that $b = 12$.

16. In each case we solve for e in terms of f:
(a) $e = f - 4$; e is to the left of f. **(b)** $e = f + 4$; e is to the right of f.
(c) $e = f + 6$; e is to the right of f. **(d)** $e = f - 7$; e is to the left of f.

17. $|6x - 2| = 7$

Case 1:
$$6x - 2 = 7,\quad 6x = 9,\quad x = 3/2$$
Case 2:
$$6x - 2 = -7,\quad 6x = -5,\quad x = -5/6$$

18. $|3 + 2x| = 11$

Case 1:
$$3 + 2x = 11,\quad 2x = 8,\quad x = 4$$
Case 2:
$$3 + 2x = -11,\quad 2x = -14,\quad x = -7$$

19. $|6x - 7| = |3 + 2x|$

Case 1:
$$6x - 7 = 3 + 2x,\quad 4x = 10,\quad x = 5/2$$
Case 2:
$$6x - 7 = -(3 + 2x),\quad 8x = 4,\quad x = 1/2$$

20. $|4x + 5| = |8x - 3|$

Case 1:
$$4x + 5 = 8x - 3,\quad -4x = -8,\quad x = 2$$
Case 2:
$$4x + 5 = -(8x - 3),\quad 12x = -2,\quad x = -1/6$$

21. $|9x| - 11 = x$

Case 1:
$$\begin{aligned} 9x - 11 &= x \\ 8x &= 11 \\ x &= 11/8 \end{aligned}$$

Case 2:
$$\begin{aligned} -9x - 11 &= x \\ -10x &= 11 \\ x &= -11/10 \end{aligned}$$

22. $2x - 7 = |x + 1|$

Case 1:
$$\begin{aligned} 2x - 7 &= x + 1 \\ x &= 8 \end{aligned}$$

Case 2:
$$\begin{aligned} 2x - 7 &= -(x + 1) \\ 3x &= 6 \\ x &= 2; \end{aligned}$$
not a solution because x must also satisfy $x < -1$

23. $\left|\dfrac{x+5}{2-x}\right| = 6$

Case 1:
$$\begin{aligned} \frac{x+5}{2-x} &= 6 \\ x + 5 &= 12 - 6x \\ 7x &= 7 \\ x &= 1 \end{aligned}$$

Case 2:
$$\begin{aligned} \frac{x+5}{2-x} &= -6 \\ x + 5 &= -12 + 6x \\ -5x &= -17 \\ x &= 17/5 \end{aligned}$$

24. $\left|\dfrac{x-3}{x+4}\right| = 5$

Case 1:
$$\begin{aligned} \frac{x-3}{x+4} &= 5 \\ x - 3 &= 5x + 20 \\ -4x &= 23 \\ x &= -23/4 \end{aligned}$$

Case 2:
$$\begin{aligned} \frac{x-3}{x+4} &= -5 \\ x - 3 &= -5x - 20 \\ 6x &= -17 \\ x &= -17/6 \end{aligned}$$

25.
$$\begin{gathered} |x + 6| < 3 \\ -3 < x + 6 < 3 \\ -9 < x < -3 \end{gathered}$$
$S = (-9, -3)$

26.
$$\begin{gathered} |7 - x| \leq 5 \\ -5 \leq 7 - x \leq 5 \\ -12 \leq -x \leq -2 \\ 12 \geq x \geq 2 \end{gathered}$$
$S = [2, 12]$

27.
$$\begin{gathered} |2x - 3| \leq 6 \\ -6 \leq 2x - 3 \leq 6 \\ -3 \leq 2x \leq 9 \\ -3/2 \leq x \leq 9/2 \end{gathered}$$
$S = [-3/2, 9/2]$

28.
$$\begin{gathered} |3x + 1| < 4 \\ -4 < 3x + 1 < 4 \\ -5 < 3x < 3 \\ -5/3 < x < 1 \end{gathered}$$
$S = (-5/3, 1)$

29. $|x + 2| > 1$

Case 1:
$$\begin{aligned} x + 2 &> 1 \\ x &> -1 \end{aligned}$$

Case 2:
$$\begin{aligned} x + 2 &< -1 \\ x &< -3 \end{aligned}$$

$S = (-\infty, -3) \cup (-1, +\infty)$

30. $\left|\dfrac{1}{2}x - 1\right| \geq 2$

Case 1:
$$\begin{aligned} \frac{1}{2}x - 1 &\geq 2 \\ \frac{1}{2}x &\geq 3 \\ x &\geq 6 \end{aligned}$$

Case 2:
$$\begin{aligned} \frac{1}{2}x - 1 &\leq -2 \\ \frac{1}{2}x &\leq -1 \\ x &\leq -2 \end{aligned}$$

$S = (-\infty, -2] \cup [6, +\infty)$

31. $|5 - 2x| \geq 4$

Case 1:
$$\begin{aligned} 5 - 2x &\geq 4 \\ -2x &\geq -1 \\ x &\leq 1/2 \end{aligned}$$

Case 2:
$$\begin{aligned} 5 - 2x &\leq -4 \\ -2x &\leq -9 \\ x &\geq 9/2 \end{aligned}$$

$S = (-\infty, 1/2] \cup [9/2, +\infty)$

32. $|7x + 1| > 3$

Case 1:
$$\begin{aligned} 7x + 1 &> 3 \\ 7x &> 2 \\ x &> 2/7 \end{aligned}$$

Case 2:
$$\begin{aligned} 7x + 1 &< -3 \\ 7x &< -4 \\ x &< -4/7 \end{aligned}$$

$S = (-\infty, -4/7) \cup (2/7, +\infty)$

33. $\dfrac{1}{|x-1|} < 2, x \neq 1$

$|x-1| > 1/2$

Case 1: $x - 1 > 1/2$, $x > 3/2$

Case 2: $x - 1 < -1/2$, $x < 1/2$

$S = (-\infty, 1/2) \cup (3/2, +\infty)$

34. $\dfrac{1}{|3x+1|} \geq 5, x \neq -1/3$

$|3x+1| \leq 1/5$

$-1/5 \leq 3x + 1 \leq 1/5$

$-6/5 \leq 3x \leq -4/5$

$-2/5 \leq x \leq -4/15$

$S = [-2/5, -1/3) \cup (-1/3, -4/15]$

35. $\dfrac{3}{|2x-1|} \geq 4, x \neq 1/2$

$\dfrac{|2x-1|}{3} \leq \dfrac{1}{4}$

$|2x-1| \leq 3/4$

$-3/4 \leq 2x - 1 \leq 3/4$

$1/4 \leq 2x \leq 7/4$

$1/8 \leq x \leq 7/8$

$S = [1/8, 1/2) \cup (1/2, 7/8]$

36. $\dfrac{2}{|x+3|} < 1, x \neq -3$

$\dfrac{|x+3|}{2} > 1$

$|x+3| > 2$

Case 1: $x + 3 > 2$, $x > -1$

Case 2: $x + 3 < -2$, $x < -5$

$S = (-\infty, -5) \cup (-1, +\infty)$

37. $\sqrt{(x^2 - 5x + 6)^2} = x^2 - 5x + 6$ if $x^2 - 5x + 6 \geq 0$ or, equivalently, if $(x-2)(x-3) \geq 0$; $x \in (-\infty, 2] \cup [3, +\infty)$.

38. If $x \geq 2$ then $3 \leq x - 2 \leq 7$ so $5 \leq x \leq 9$; if $x < 2$ then $3 \leq 2 - x \leq 7$ so $-5 \leq x \leq -1$. $S = [-5, -1] \cup [5, 9]$.

39. If $u = |x-3|$ then $u^2 - 4u = 12$, $u^2 - 4u - 12 = 0$, $(u-6)(u+2) = 0$, so $u = 6$ or $u = -2$. If $u = 6$ then $|x-3| = 6$, so $x = 9$ or $x = -3$. If $u = -2$ then $|x-3| = -2$ which is impossible. The solutions are -3 and 9.

41.
$$\begin{aligned} |a-b| &= |a + (-b)| \\ &\leq |a| + |-b| \text{ (triangle inequality)} \\ &= |a| + |b|. \end{aligned}$$

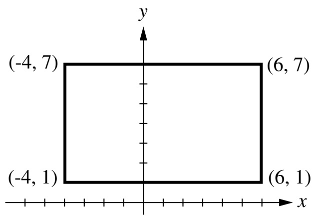
42.
$$\begin{aligned} a &= (a-b) + b \\ |a| &= |(a-b) + b| \\ |a| &\leq |a-b| + |b| \text{ (triangle inequality)} \\ |a| - |b| &\leq |a-b|. \end{aligned}$$

43. From Exercise 42
(i) $|a| - |b| \leq |a-b|$; but $|b| - |a| \leq |b-a| = |a-b|$, so (ii) $|a| - |b| \geq -|a-b|$.
Combining (i) and (ii): $-|a-b| \leq |a| - |b| \leq |a-b|$, so $||a| - |b|| \leq |a-b|$.

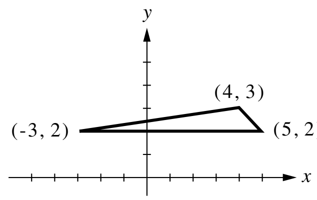
APPENDIX C
Coordinate Planes and Lines

EXERCISE SET C

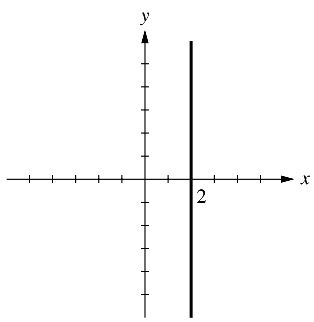
1.



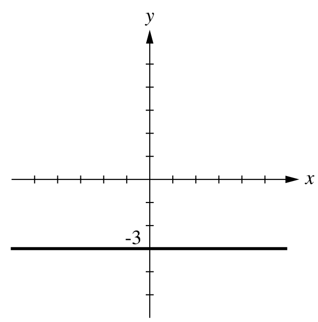
2. area $= \frac{1}{2}bh = \frac{1}{2}(5-(-3))(1) = 4$



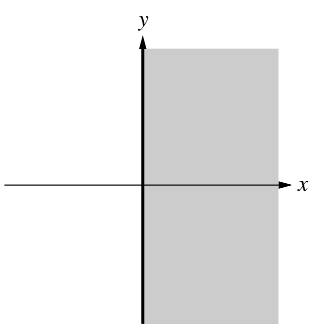
3. **(a)** $x = 2$



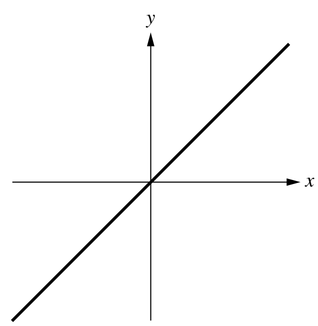
(b) $y = -3$



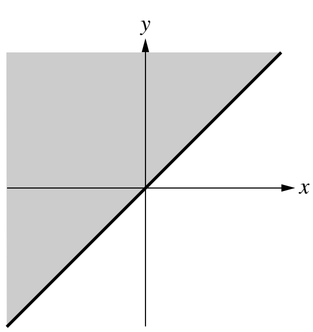
(c) $x \geq 0$



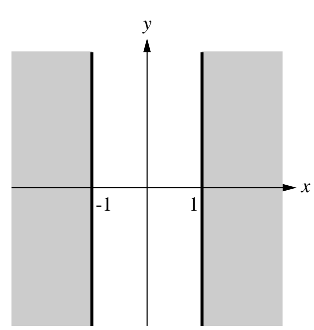
(d) $y = x$



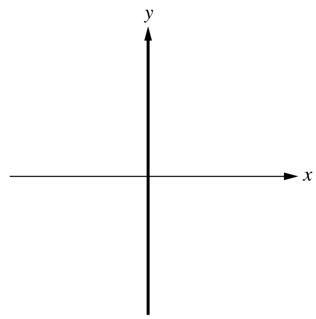
(e) $y \geq x$



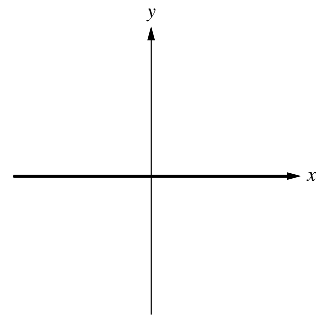
(f) $|x| \geq 1$



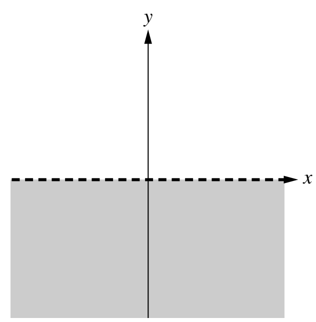
4. **(a)** $x = 0$



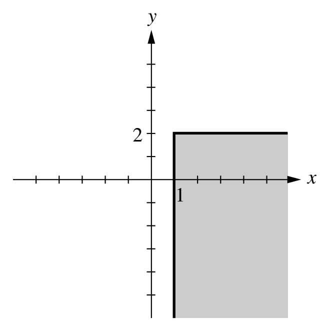
(b) $y = 0$



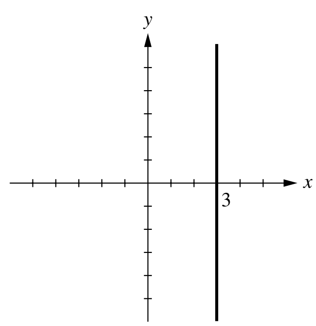
(c) $y < 0$



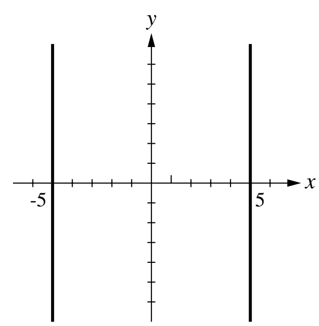
(d) $x \geq 1$ and $y \leq 2$



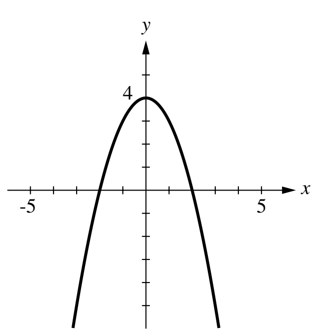
(e) $x = 3$



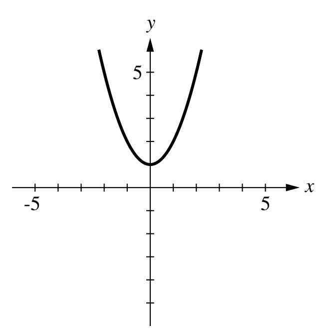
(f) $|x| = 5$



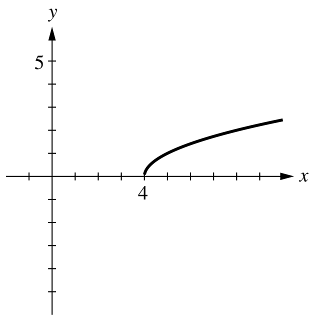
5. $y = 4 - x^2$



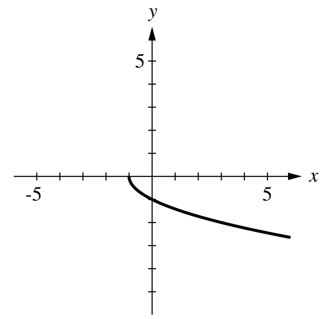
6. $y = 1 + x^2$



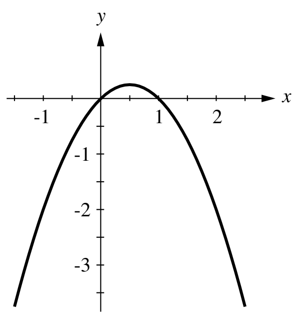
7. $y = \sqrt{x-4}$



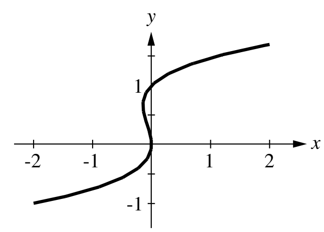
8. $y = -\sqrt{x+1}$



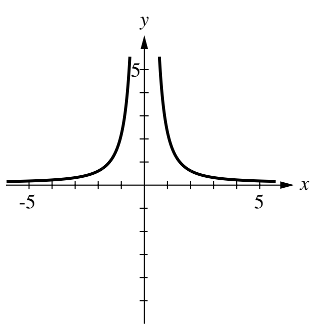
9. $x^2 - x + y = 0$



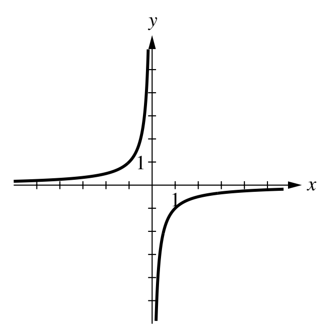
10. $x = y^3 - y^2$



11. $x^2 y = 2$



12. $xy = -1$



13. **(a)** $m = \dfrac{4-2}{3-(-1)} = \dfrac{1}{2}$ **(b)** $m = \dfrac{1-3}{7-5} = -1$

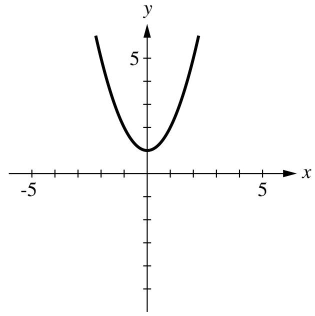
(c) $m = \dfrac{\sqrt{2}-\sqrt{2}}{-3-4} = 0$ **(d)** $m = \dfrac{12-(-6)}{-2-(-2)} = \dfrac{18}{0}$, not defined

14. $m_1 = \dfrac{5-2}{6-(-1)} = \dfrac{3}{7}$, $m_2 = \dfrac{7-2}{2-(-1)} = \dfrac{5}{3}$, $m_3 = \dfrac{7-5}{2-6} = -\dfrac{1}{2}$

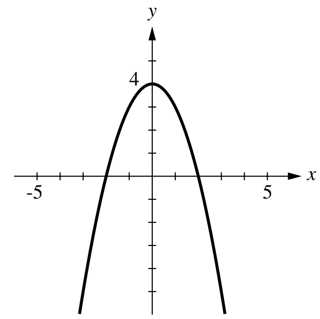
15. **(a)** The line through $(1,1)$ and $(-2,-5)$ has slope $m_1 = \dfrac{-5-1}{-2-1} = 2$, the line through $(1,1)$ and $(0,-1)$ has slope $m_2 = \dfrac{-1-1}{0-1} = 2$. The given points lie on a line because $m_1 = m_2$.

(b) The line through $(-2,4)$ and $(0,2)$ has slope $m_1 = \dfrac{2-4}{0+2} = -1$, the line through $(-2,4)$ and $(1,5)$ has slope $m_2 = \dfrac{5-4}{1+2} = \dfrac{1}{3}$. The given points do not lie on a line because $m_1 \neq m_2$.

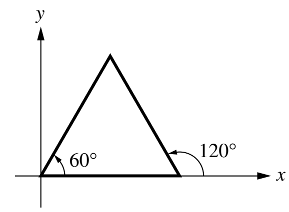
16.



17.



18. The triangle is equiangular because it is equilateral. The angles of inclination of the sides are $0°$, $60°$, and $120°$ (see figure), thus the slopes of its sides are $\tan 0° = 0$, $\tan 60° = \sqrt{3}$, and $\tan 120° = -\sqrt{3}$.



19. III < II < IV < I

20. III < IV < I < II

21. Use the points $(1, 2)$ and (x, y) to calculate the slope: $(y - 2)/(x - 1) = 3$
(a) if $x = 5$, then $(y - 2)/(5 - 1) = 3$, $y - 2 = 12$, $y = 14$
(b) if $y = -2$, then $(-2 - 2)/(x - 1) = 3$, $x - 1 = -4/3$, $x = -1/3$

22. Use $(7, 5)$ and (x, y) to calculate the slope: $(y - 5)/(x - 7) = -2$
(a) if $x = 9$, then $(y - 5)/(9 - 7) = -2$, $y - 5 = -4$, $y = 1$
(b) if $y = 12$, then $(12 - 5)/(x - 7) = -2$, $x - 7 = -7/2$, $x = 7/2$

23. Using $(3, k)$ and $(-2, 4)$ to calculate the slope, we find $\dfrac{k - 4}{3 - (-2)} = 5$, $k - 4 = 25$, $k = 29$.

24. The slope obtained by using the points $(1, 5)$ and $(k, 4)$ must be the same as that obtained from the points $(1, 5)$ and $(2, -3)$ so $\dfrac{4 - 5}{k - 1} = \dfrac{-3 - 5}{2 - 1}$, $-\dfrac{1}{k - 1} = -8$, $k - 1 = 1/8$, $k = 9/8$.

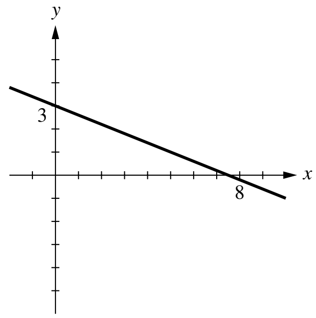
25. $\dfrac{0 - 2}{x - 1} = -\dfrac{0 - 5}{x - 4}$, $-2x + 8 = 5x - 5$, $7x = 13$, $x = 13/7$

26. Use $(0, 0)$ and (x, y) to get $\dfrac{y - 0}{x - 0} = \dfrac{1}{2}$, $y = \dfrac{1}{2}x$. Use $(7, 5)$ and (x, y) to get $\dfrac{y - 5}{x - 7} = 2$, $y - 5 = 2(x - 7)$, $y = 2x - 9$. Solve the system of equations $y = \dfrac{1}{2}x$ and $y = 2x - 9$ to get $x = 6$, $y = 3$.

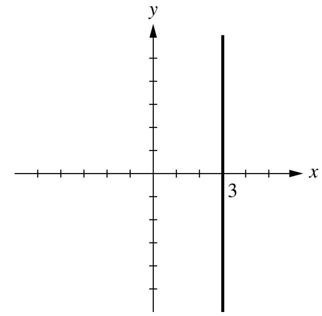
27. Show that opposite sides are parallel by showing that they have the same slope:
using $(3, -1)$ and $(6, 4)$, $m_1 = 5/3$; using $(6, 4)$ and $(-3, 2)$, $m_2 = 2/9$;
using $(-3, 2)$ and $(-6, -3)$, $m_3 = 5/3$; using $(-6, -3)$ and $(3, -1)$, $m_4 = 2/9$.
Opposite sides are parallel because $m_1 = m_3$ and $m_2 = m_4$.

28. The line through $(3, 1)$ and $(6, 3)$ has slope $m_1 = 2/3$, the line through $(3, 1)$ and $(2, 9)$ has slope $m_2 = -8$, the line through $(6, 3)$ and $(2, 9)$ has slope $m_3 = -3/2$. Because $m_1 m_3 = -1$, the corresponding lines are perpendicular so the given points are vertices of a right triangle.

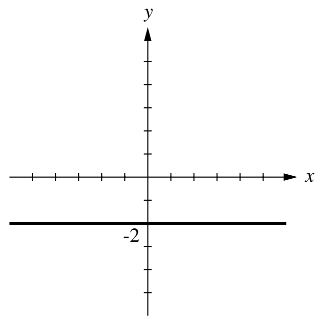
29. (a)



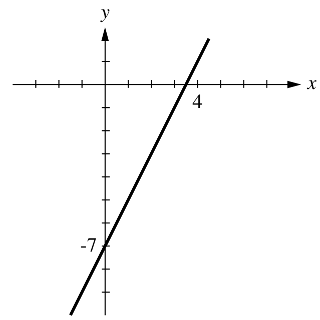
(b)



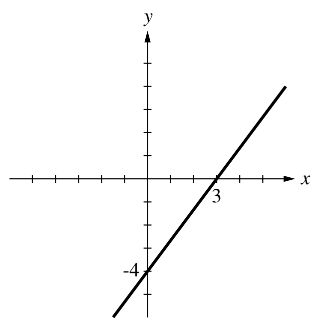
(c)



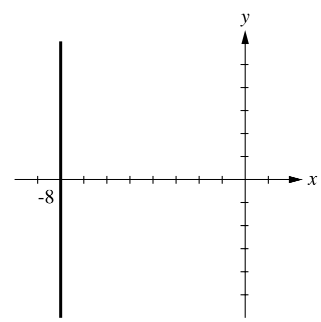
(d)



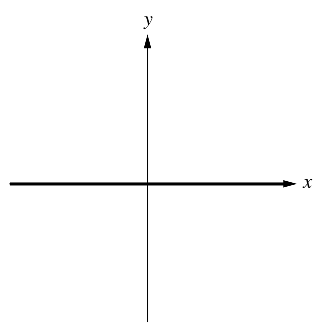
30. (a)



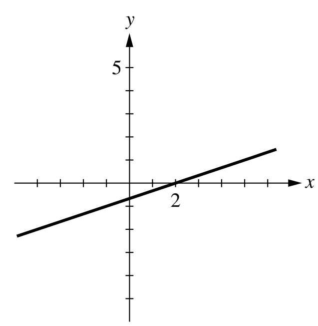
(b)



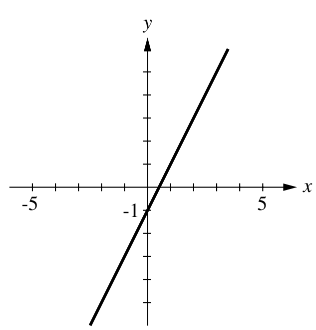
(c)



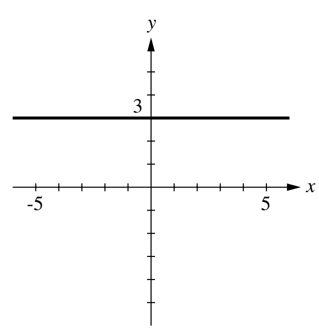
(d)



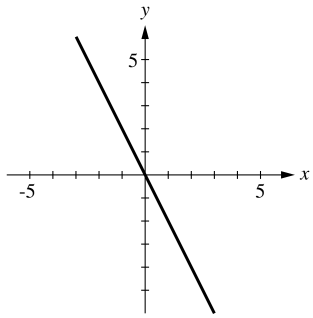
31. (a)



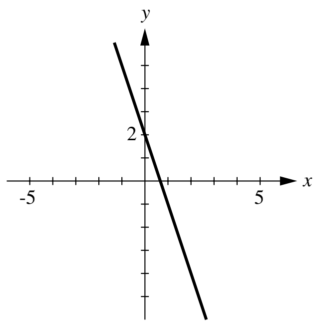
(b)



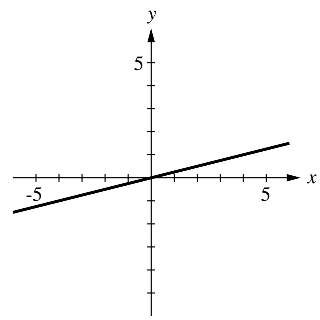
(c)



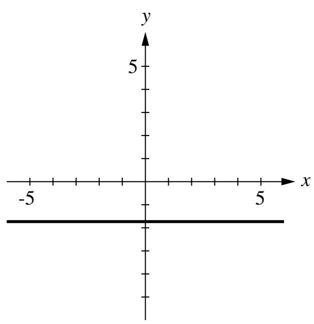
32. **(a)**



(b)



(c)



33. **(a)** $m = 3$, $b = 2$

(b) $m = -\dfrac{1}{4}$, $b = 3$

(c) $y = -\dfrac{3}{5}x + \dfrac{8}{5}$ so $m = -\dfrac{3}{5}$, $b = \dfrac{8}{5}$

(d) $m = 0$, $b = 1$

(e) $y = -\dfrac{b}{a}x + b$ so $m = -\dfrac{b}{a}$, y-intercept b

34. **(a)** $m = -4$, $b = 2$

(b) $y = \dfrac{1}{3}x - \dfrac{2}{3}$ so $m = \dfrac{1}{3}$, $b = -\dfrac{2}{3}$

(c) $y = -\dfrac{3}{2}x + 3$ so $m = -\dfrac{3}{2}$, $b = 3$

(d) $y = 3$ so $m = 0$, $b = 3$

(e) $y = -\dfrac{a_0}{a_1}x$ so $m = -\dfrac{a_0}{a_1}$, $b = 0$

35. **(a)** $m = (0 - (-3))/(2 - 0)) = 3/2$ so $y = 3x/2 - 3$
(b) $m = (-3 - 0)/(4 - 0) = -3/4$ so $y = -3x/4$

36. **(a)** $m = (0 - 2)/(2 - 0)) = -1$ so $y = -x + 2$
(b) $m = (2 - 0)/(3 - 0) = 2/3$ so $y = 2x/3$

37. $y = -2x + 4$

38. $y = 5x - 3$

39. The slope m of the line must equal the slope of $y = 4x-2$, thus $m = 4$ so the equation is $y = 4x+7$.

40. The slope of the line $3x + 2y = 5$ is $-3/2$ so the line through $(-1, 2)$ with this slope is $y - 2 = -\frac{3}{2}(x + 1)$; $y = -\frac{3}{2}x + \frac{1}{2}$.

41. The slope m of the line must be the negative reciprocal of the slope of $y = 5x + 9$, thus $m = -1/5$ and the equation is $y = -x/5 + 6$.

42. The slope of the line $x - 4y = 7$ is $1/4$ so a line perpendicular to it must have a slope of -4; $y + 4 = -4(x - 3)$; $y = -4x + 8$.

43. $y - 4 = \dfrac{-7 - 4}{1 - 2}(x - 2) = 11(x - 2)$, $y = 11x - 18$.

44. $y - 6 = \dfrac{1 - 6}{-2 - (-3)}(x - (-3))$, $y - 6 = -5(x + 3)$, $y = -5x - 9$.

45. The line passes through $(0, 2)$ and $(-4, 0)$, thus $m = \dfrac{0 - 2}{-4 - 0} = \dfrac{1}{2}$ so $y = \dfrac{1}{2}x + 2$.

46. The line passes through $(0, b)$ and $(a, 0)$, thus $m = \dfrac{0 - b}{a - 0} = -\dfrac{b}{a}$, so the equation is $y = -\dfrac{b}{a}x + b$.

47. $y = 1$

48. $y = -8$

49. **(a)** $m_1 = 4$, $m_2 = 4$; parallel because $m_1 = m_2$

(b) $m_1 = 2$, $m_2 = -1/2$; perpendicular because $m_1 m_2 = -1$

(c) $m_1 = 5/3$, $m_2 = 5/3$; parallel because $m_1 = m_2$

(d) If $A \neq 0$ and $B \neq 0$, then $m_1 = -A/B$, $m_2 = B/A$ and the lines are perpendicular because $m_1 m_2 = -1$. If either A or B (but not both) is zero, then the lines are perpendicular because one is horizontal and the other is vertical.

(e) $m_1 = 4$, $m_2 = 1/4$; neither

50. **(a)** $m_1 = -5$, $m_2 = -5$; parallel because $m_1 = m_2$

(b) $m_1 = 2$, $m_2 = -1/2$; perpendicular because $m_1 m_2 = -1$.

(c) $m_1 = -4/5$, $m_2 = 5/4$; perpendicular because $m_1 m_2 = -1$.

(d) If $B \neq 0$, then $m_1 = m_2 = -A/B$ and the lines are parallel because $m_1 = m_2$. If $B = 0$ (and $A \neq 0$), then the lines are parallel because they are both perpendicular to the x-axis.

(e) $m_1 = 1/2$, $m_2 = 2$; neither

51. $y = (-3/k)x + 4/k$, $k \neq 0$

(a) $-3/k = 2$, $k = -3/2$

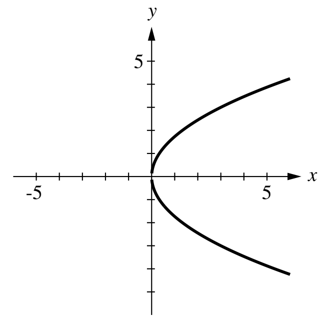
(b) $4/k = 5$, $k = 4/5$

(c) $3(-2) + k(4) = 4$, $k = 5/2$

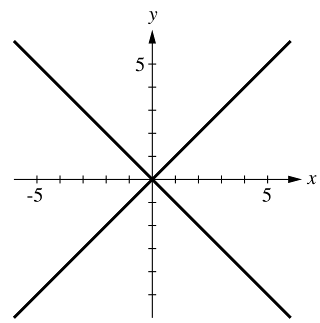
(d) The slope of $2x - 5y = 1$ is $2/5$ so $-3/k = 2/5$, $k = -15/2$.

(e) The slope of $4x + 3y = 2$ is $-4/3$ so the slope of a line perpendicular to it is $3/4$; $-3/k = 3/4$, $k = -4$.

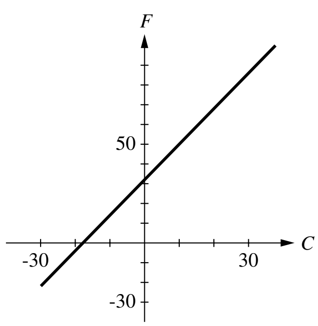
52. $y^2 = 3x$: the union of the graphs of $y = \sqrt{3x}$ and $y = -\sqrt{3x}$



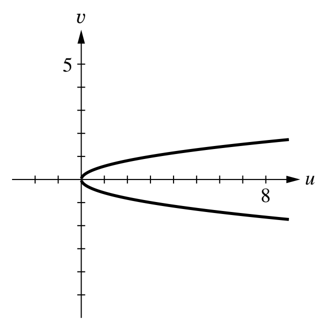
53. $(x - y)(x + y) = 0$: the union of the graphs of $x - y = 0$ and $x + y = 0$



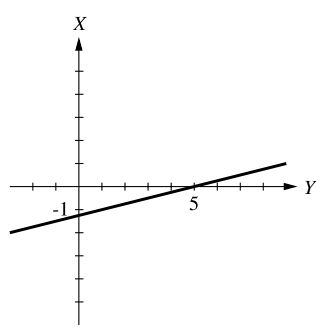
54. $F = \dfrac{9}{5}C + 32$



55. $u = 3v^2$



56. $Y = 4X + 5$

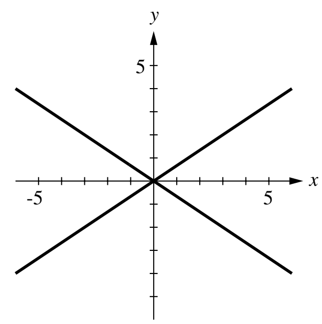


57. Solve $x = 5t + 2$ for t to get $t = \dfrac{1}{5}x - \dfrac{2}{5}$, so $y = \left(\dfrac{1}{5}x - \dfrac{2}{5}\right) - 3 = \dfrac{1}{5}x - \dfrac{17}{5}$, which is a line.

58. Solve $x = 1 + 3t^2$ for t^2 to get $t^2 = \dfrac{1}{3}x - \dfrac{1}{3}$, so $y = 2 - \left(\dfrac{1}{3}x - \dfrac{1}{3}\right) = -\dfrac{1}{3}x + \dfrac{7}{3}$, which is a line; $1 + 3t^2 \geq 1$ for all t so $x \geq 1$.

59. An equation of the line through $(1, 4)$ and $(2, 1)$ is $y = -3x + 7$. It crosses the y-axis at $y = 7$, and the x-axis at $x = 7/3$, so the area of the triangle is $\dfrac{1}{2}(7)(7/3) = 49/6$.

60. $(2x - 3y)(2x + 3y) = 0$, so $2x - 3y = 0$, $y = \dfrac{2}{3}x$ or $2x + 3y = 0$, $y = -\dfrac{2}{3}x$. The graph consists of the lines $y = \pm\dfrac{2}{3}x$.



61. **(a)** yes **(b)** yes **(c)** no **(d)** yes
(e) yes **(f)** yes **(g)** no

APPENDIX D
Distances, Circles, and Quadratic Equations

EXERCISE SET D

1. in the proof of Theorem D.1

2. **(a)** $d = \sqrt{(-1-2)^2 + (1-5)^2} = \sqrt{9+16} = \sqrt{25} = 5$

(b) $\left(\dfrac{2+(-1)}{2}, \dfrac{5+1}{2}\right) = (1/2, 3)$

3. **(a)** $d = \sqrt{(1-7)^2 + (9-1)^2} = \sqrt{36+64} = \sqrt{100} = 10$

(b) $\left(\dfrac{7+1}{2}, \dfrac{1+9}{2}\right) = (4, 5)$

4. **(a)** $d = \sqrt{(-3-2)^2 + (6-0)^2} = \sqrt{25+36} = \sqrt{61}$

(b) $\left(\dfrac{2+(-3)}{2}, \dfrac{0+6}{2}\right) = (-1/2, 3)$

5. **(a)** $d = \sqrt{[-7-(-2)]^2 + [-4-(-6)]^2} = \sqrt{25+4} = \sqrt{29}$

(b) $\left(\dfrac{-2+(-7)}{2}, \dfrac{-6+(-4)}{2}\right) = (-9/2, -5)$

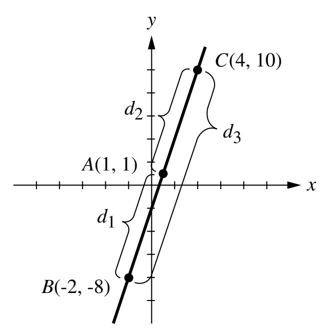
6. Let $A(1,1)$, $B(-2,-8)$, and $C(4,10)$ be the given points (see diagram). A, B, and C lie on a straight line if and only if $d_1 + d_2 = d_3$, where d_1, d_2, and d_3 are the lengths of the line segments AB, AC, and BC. But

$d_1 = \sqrt{(-2-1)^2 + (-8-1)^2} = 3\sqrt{10}$,

$d_2 = \sqrt{(4-1)^2 + (10-1)^2} = 3\sqrt{10}$,

$d_3 = \sqrt{(4+2)^2 + (10+8)^2} = 6\sqrt{10}$; because $d_1 + d_2 = d_3$,

it follows that A, B, and C lie on a straight line.



7. Let $A(5,-2)$, $B(6,5)$, and $C(2,2)$ be the given vertices and a, b, and c the lengths of the sides opposite these vertices; then

$a = \sqrt{(2-6)^2 + (2-5)^2} = \sqrt{25} = 5$ and $b = \sqrt{(2-5)^2 + (2+2)^2} = \sqrt{25} = 5$.

Triangle ABC is isosceles because it has two equal sides ($a = b$).

8. A triangle is a right triangle if and only if the square of the longest side is equal to the sum of the squares of the other two sides (Pythagorean theorem). With $A(1,3)$, $B(4,2)$, and $C(-2,-6)$ as vertices and s_1, s_2, and s_3 the lengths of the sides opposite these vertices we find that $s_1^2 = (-2-4)^2 + (-6-2)^2 = 100$, $s_2^2 = (-2-1)^2 + (-6-3)^2 = 90$, $s_3^2 = (4-1)^2 + (2-3)^2 = 10$, and that $s_1^2 = s_2^2 + s_3^2$, so ABC is a right triangle. The right angle occurs at the vertex $A(1,3)$.

9. $P_1(0,-2)$, $P_2(-4,8)$, and $P_3(3,1)$ all lie on a circle whose center is $C(-2,3)$ if the points P_1, P_2 and P_3 are equidistant from C. Denoting the distances between P_1, P_2, P_3 and C by d_1, d_2 and d_3 we find that $d_1 = \sqrt{(0+2)^2 + (-2-3)^2} = \sqrt{29}$, $d_2 = \sqrt{(-4+2)^2 + (8-3)^2} = \sqrt{29}$, and $d_3 = \sqrt{(3+2)^2 + (1-3)^2} = \sqrt{29}$, so P_1, P_2 and P_3 lie on a circle whose center is $C(-2,3)$ because $d_1 = d_2 = d_3$.

10. The distance between $(t, 2t-6)$ and $(0,4)$ is
$\sqrt{(t-0)^2+(2t-6-4)^2}=\sqrt{t^2+(2t-10)^2}=\sqrt{5t^2-40t+100}$;
the distance between $(t, 2t-6)$ and $(8,0)$ is $\sqrt{(t-8)^2+(2t-6)^2}=\sqrt{5t^2-40t+100}$,
so $(t, 2t-6)$ is equidistant from $(0,4)$ and $(8,0)$.

11. If $(2,k)$ is equidistant from (3,7) and (9,1), then
$\sqrt{(2-3)^2+(k-7)^2}=\sqrt{(2-9)^2+(k-1)^2}$, $1+(k-7)^2=49+(k-1)^2$,
$1+k^2-14k+49=49+k^2-2k+1$, $-12k=0$, $k=0$.

12. $(x-3)/2=4$ and $(y+2)/2=-5$ so $x=11$ and $y=-12$.

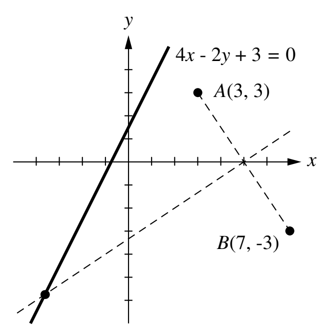
13. The slope of the line segment joining $(2,8)$ and $(-4,6)$ is $\dfrac{6-8}{-4-2}=\dfrac{1}{3}$ so the slope of the perpendicular bisector is -3. The midpoint of the line segment is $(-1,7)$ so an equation of the bisector is $y-7=-3(x+1)$; $y=-3x+4$.

14. The slope of the line segment joining $(5,-1)$ and $(4,8)$ is $\dfrac{8-(-1)}{4-5}=-9$ so the slope of the perpendicular bisector is $\dfrac{1}{9}$. The midpoint of the line segment is $(9/2, 7/2)$ so an equation of the bisector is $y-\dfrac{7}{2}=\dfrac{1}{9}\left(x-\dfrac{9}{2}\right)$; $y=\dfrac{1}{9}x+3$.

15. Method (see figure): Find an equation of the perpendicular bisector of the line segment joining $A(3,3)$ and $B(7,-3)$. All points on this perpendicular bisector are equidistant from A and B, thus find where it intersects the given line.

The midpoint of AB is $(5,0)$, the slope of AB is $-3/2$ thus the slope of the perpendicular bisector is $2/3$ so an equation is

$$\begin{aligned} y-0&=\frac{2}{3}(x-5)\\ 3y&=2x-10\\ 2x-3y-10&=0. \end{aligned}$$



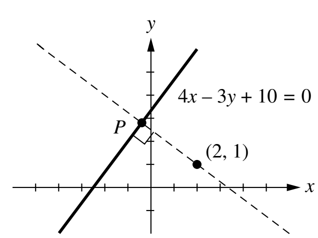
The solution of the system

$$\begin{cases} 4x-2y+3=0\\ 2x-3y-10=0 \end{cases}$$

gives the point $(-29/8, -23/4)$.

16. **(a)** $y=4$ is a horizontal line, so the vertical distance is $|4-(-2)|=|6|=6$.
(b) $x=-1$ is a vertical line, so the horizontal distance is $|-1-3|=|-4|=4$.

17. Method (see figure): write an equation of the line that goes through the given point and that is perpendicular to the given line; find the point P where this line intersects the given line; find the distance between P and the given point.



The slope of the given line is $4/3$, so the slope of a line perpendicular to it is $-3/4$.

The line through $(2,1)$ having a slope of $-3/4$ is $y-1=-\dfrac{3}{4}(x-2)$ or, after simplification, $3x+4y=10$ which when solved simultaneously with $4x-3y+10=0$ yields $(-2/5,14/5)$ as the point of intersection. The distance d between $(-2/5,14/5)$ and $(2,1)$ is $d=\sqrt{(2+2/5)^2+(1-14/5)^2}=3$.

18. (See the solution to Exercise 17 for a description of the method.) The slope of the line $5x+12y-36=0$ is $-5/12$. The line through $(8,4)$ and perpendicular to the given line is $y-4=\dfrac{12}{5}(x-8)$ or, after simplification, $12x-5y=76$. The point of intersection of this line with the given line is found to be $\left(\dfrac{84}{13},\dfrac{4}{13}\right)$ and the distance between it and $(8,4)$ is 4.

19. If $B=0$, then the line $Ax+C=0$ is vertical and $x=-C/A$ for each point on the line. The line through (x_0,y_0) and perpendicular to the given line is horizontal and intersects the given line at the point $(-C/A,\ y_0)$. The distance d between $(-C/A,\ y_0)$ and (x_0,y_0) is

$$d=\sqrt{(x_0+C/A)^2+(y_0-y_0)^2}=\sqrt{\frac{(Ax_0+C)^2}{A^2}}=\frac{|Ax_0+C|}{\sqrt{A^2}}$$

which is the value of $\dfrac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$ for $B=0$.

If $B\neq 0$, then the slope of the given line is $-A/B$ and the line through (x_0,y_0) and perpendicular to the given line is

$y-y_0=\dfrac{B}{A}(x-x_0)$, $Ay-Ay_0=Bx-Bx_0$, $Bx-Ay=Bx_0-Ay_0$.

The point of intersection of this line and the given line is obtained by solving

$Ax+By=-C$ and $Bx-Ay=Bx_0-Ay_0$.

Multiply the first equation through by A and the second by B and add the results to get

$(A^2+B^2)x=B^2x_0-ABy_0-AC$ so $x=\dfrac{B^2x_0-ABy_0-AC}{A^2+B^2}$

Similarly, by multiplying by B and $-A$, we get $y=\dfrac{-ABx_0+A^2y_0-BC}{A^2+B^2}$.

The square of the distance d between (x,y) and (x_0,y_0) is

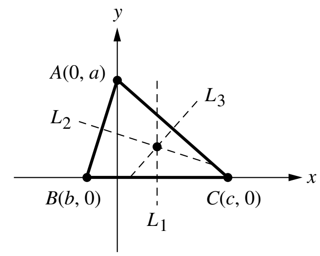
$$\begin{aligned}
d^2&=\left[x_0-\frac{B^2x_0-ABy_0-AC}{A^2+B^2}\right]^2+\left[y_0-\frac{-ABx_0+A^2y_0-BC}{A^2+B^2}\right]^2\\
&=\frac{(A^2x_0+ABy_0+AC)^2}{(A^2+B^2)^2}+\frac{(ABx_0+B^2y_0+BC)^2}{(A^2+B^2)^2}\\
&=\frac{A^2(Ax_0+By_0+C)^2+B^2(Ax_0+By_0+C)^2}{(A^2+B^2)^2}\\
&=\frac{(Ax_0+By_0+C)^2(A^2+B^2)}{(A^2+B^2)^2}=\frac{(Ax_0+By_0+C)^2}{A^2+B^2}
\end{aligned}$$

so $d=\dfrac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$.

20. $d=\dfrac{|4(2)-3(1)+10|}{\sqrt{4^2+(-3)^2}}=\dfrac{|15|}{\sqrt{25}}=\dfrac{15}{5}=3.$

21. $d=\dfrac{|5(8)+12(4)-36|}{\sqrt{5^2+12^2}}=\dfrac{|52|}{\sqrt{169}}=\dfrac{52}{13}=4.$

22. Method (see figure): Let $A(0, a)$, $B(b, 0)$, and $C(c, 0)$ be the given vertices; find equations for the perpendicular bisectors L_1, L_2, and L_3 and show that they all intersect at the same point.



line L_1: The midpoint of BC is $\left(\dfrac{b+c}{2}, 0\right)$ and since L_1 is vertical, an equation for L_1 is $x = \dfrac{b+c}{2}$;

line L_2: The midpoint of AB is $\left(\dfrac{b}{2}, \dfrac{a}{2}\right)$; the slope of AB is $-\dfrac{a}{b}$ (if $b \neq 0$) so the slope of L_2 is $\dfrac{b}{a}$ (even if $b = 0$) and an equation of L_2 is $y - \dfrac{a}{2} = \dfrac{b}{a}\left(x - \dfrac{b}{2}\right)$;

line L_3: The midpoint of AC is $\left(\dfrac{c}{2}, \dfrac{a}{2}\right)$; the slope of AC is $-\dfrac{a}{c}$ (if $c \neq 0$) so the slope of L_3 is $\dfrac{c}{a}$ (even if $c = 0$) and an equation of L_3 is $y - \dfrac{a}{2} = \dfrac{c}{a}\left(x - \dfrac{c}{2}\right)$.

For the point of intersection of L_1 and L_2, solve $x = \dfrac{b+c}{2}$ and $y - \dfrac{a}{2} = \dfrac{b}{a}\left(x - \dfrac{b}{2}\right)$.

The point is found to be $\left(\dfrac{b+c}{2}, \dfrac{a^2+bc}{2a}\right)$. The point of intersection of L_1 and L_3 is obtained by solving the system $x = \dfrac{b+c}{2}$ and $y - \dfrac{a}{2} = \dfrac{c}{a}\left(x - \dfrac{c}{2}\right)$, its solution yields the point $\left(\dfrac{b+c}{2}, \dfrac{a^2+bc}{2a}\right)$.

So L_1, L_2, and L_3 all intersect at the same point.

23. **(a)** center (0,0), radius 5 **(b)** center (1,4), radius 4
(c) center $(-1, -3)$, radius $\sqrt{5}$ **(d)** center $(0, -2)$, radius 1

24. **(a)** center $(0, 0)$, radius 3 **(b)** center $(3, 5)$, radius 6
(c) center $(-4, -1)$, radius $\sqrt{8}$ **(d)** center $(-1, 0)$, radius 1

25. $(x-3)^2 + (y-(-2))^2 = 4^2$, $(x-3)^2 + (y+2)^2 = 16$

26. $(x-1)^2 + (y-0)^2 = (\sqrt{8}/2)^2$, $(x-1)^2 + y^2 = 2$

27. $r = 8$ because the circle is tangent to the x-axis, so $(x+4)^2 + (y-8)^2 = 64$.

28. $r = 5$ because the circle is tangent to the y-axis, so $(x-5)^2 + (y-8)^2 = 25$.

29. $(0, 0)$ is on the circle, so $r = \sqrt{(-3-0)^2 + (-4-0)^2} = 5$; $(x+3)^2 + (y+4)^2 = 25$.

30. $r = \sqrt{(4-1)^2 + (-5-3)^2} = \sqrt{73}$; $(x-4)^2 + (y+5)^2 = 73$.

31. The center is the midpoint of the line segment joining $(2, 0)$ and $(0, 2)$ so the center is at $(1, 1)$. The radius is $r = \sqrt{(2-1)^2 + (0-1)^2} = \sqrt{2}$, so $(x-1)^2 + (y-1)^2 = 2$.

32. The center is the midpoint of the line segment joining $(6, 1)$ and $(-2, 3)$, so the center is at $(2, 2)$. The radius is $r = \sqrt{(6-2)^2 + (1-2)^2} = \sqrt{17}$, so $(x-2)^2 + (y-2)^2 = 17$.

33. $(x^2 - 2x) + (y^2 - 4y) = 11$, $(x^2 - 2x + 1) + (y^2 - 4y + 4) = 11 + 1 + 4$, $(x-1)^2 + (y-2)^2 = 16$; center (1,2) and radius 4

34. $(x^2+8x)+y^2=-8$, $(x^2+8x+16)+y^2=-8+16$, $(x+4)^2+y^2=8$; center $(-4,0)$ and radius $2\sqrt{2}$

35. $2(x^2+2x)+2(y^2-2y)=0$, $2(x^2+2x+1)+2(y^2-2y+1)=2+2$, $(x+1)^2+(y-1)^2=2$; center $(-1,1)$ and radius $\sqrt{2}$

36. $6(x^2-x)+6(y^2+y)=3$, $6(x^2-x+1/4)+6(y^2+y+1/4)=3+6/4+6/4$, $(x-1/2)^2+(y+1/2)^2=1$; center $(1/2,-1/2)$ and radius 1

37. $(x^2+2x)+(y^2+2y)=-2$, $(x^2+2x+1)+(y^2+2y+1)=-2+1+1$, $(x+1)^2+(y+1)^2=0$; the point $(-1,-1)$

38. $(x^2-4x)+(y^2-6y)=-13$, $(x^2-4x+4)+(y^2-6y+9)=-13+4+9$, $(x-2)^2+(y-3)^2=0$; the point $(2,3)$

39. $x^2+y^2=1/9$; center $(0,0)$ and radius $1/3$

40. $x^2+y^2=4$; center (0,0) and radius 2

41. $x^2+(y^2+10y)=-26$, $x^2+(y^2+10y+25)=-26+25$, $x^2+(y+5)^2=-1$; no graph

42. $(x^2-10x)+(y^2-2y)=-29$, $(x^2-10x+25)+(y^2-2y+1)=-29+25+1$, $(x-5)^2+(y-1)^2=-3$; no graph

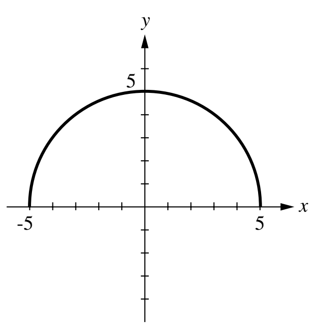
43. $16\left(x^2+\frac{5}{2}x\right)+16(y^2+y)=7$, $16\left(x^2+\frac{5}{2}x+\frac{25}{16}\right)+16\left(y^2+y+\frac{1}{4}\right)=7+25+4$, $(x+5/4)^2+(y+1/2)^2=9/4$; center $(-5/4,-1/2)$ and radius $3/2$

44. $4(x^2-4x)+4(y^2-6y)=9$, $4(x^2-4x+4)+4(y^2-6y+9)=9+16+36$, $(x-2)^2+(y-3)^2=61/4$; center $(2,3)$ and radius $\sqrt{61}/2$

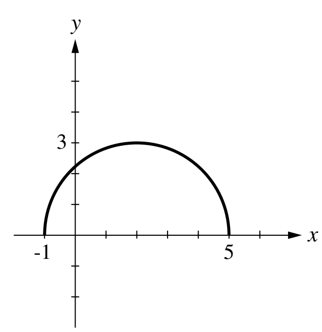
45. **(a)** $y^2=16-x^2$, so $y=\pm\sqrt{16-x^2}$. The bottom half is $y=-\sqrt{16-x^2}$.
(b) Complete the square in y to get $(y-2)^2=3-2x-x^2$, so $y-2=\pm\sqrt{3-2x-x^2}$, or $y=2\pm\sqrt{3-2x-x^2}$. The top half is $y=2+\sqrt{3-2x-x^2}$.

46. **(a)** $x^2=9-y^2$ so $x=\pm\sqrt{9-y^2}$. The right half is $x=\sqrt{9-y^2}$.
(b) Complete the square in x to get $(x-2)^2=1-y^2$ so $x-2=\pm\sqrt{1-y^2}$, $x=2\pm\sqrt{1-y^2}$. The left half is $x=2-\sqrt{1-y^2}$.

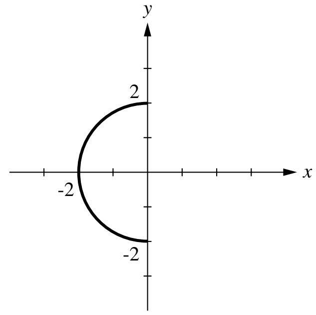
47. **(a)**



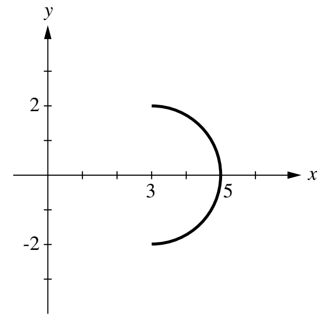
(b)
$$\begin{aligned} y &= \sqrt{5+4x-x^2} \\ &= \sqrt{5-(x^2-4x)} \\ &= \sqrt{5+4-(x^2-4x+4)} \\ &= \sqrt{9-(x-2)^2} \end{aligned}$$



48. **(a)**



(b)



49. The tangent line is perpendicular to the radius at the point. The slope of the radius is $4/3$, so the slope of the perpendicular is $-3/4$. An equation of the tangent line is $y-4=-\dfrac{3}{4}(x-3)$, or $y=-\dfrac{3}{4}x+\dfrac{25}{4}$.

50. **(a)** $(x+1)^2+y^2=10$, center at $C(-1,0)$. The slope of CP is $-1/3$ so the slope of the tangent is 3; $y+1=3(x-2)$, $y=3x-7$.

(b) $(x-3)^2+(y+2)^2=26$, center at $C(3,-2)$. The slope of CP is 5 so the slope of the tangent is $-\dfrac{1}{5}$; $y-3=-\dfrac{1}{5}(x-4)$, $y=-\dfrac{1}{5}x+\dfrac{19}{5}$.

51. **(a)** The center of the circle is at (0,0) and its radius is $\sqrt{20}=2\sqrt{5}$. The distance between P and the center is $\sqrt{(-1)^2+(2)^2}=\sqrt{5}$ which is less than $2\sqrt{5}$, so P is inside the circle.

(b) Draw the diameter of the circle that passes through P, then the shorter segment of the diameter is the shortest line that can be drawn from P to the circle, and the longer segment is the longest line that can be drawn from P to the circle (can you prove it?). Thus, the smallest distance is $2\sqrt{5}-\sqrt{5}=\sqrt{5}$, and the largest is $2\sqrt{5}+\sqrt{5}=3\sqrt{5}$.

52. **(a)** $x^2+(y-1)^2=5$, center at $C(0,1)$ and radius $\sqrt{5}$. The distance between P and C is $3\sqrt{5}/2$ so P is outside the circle.

(b) The smallest distance is $\dfrac{3}{2}\sqrt{5}-\sqrt{5}=\dfrac{1}{2}\sqrt{5}$, the largest distance is $\dfrac{3}{2}\sqrt{5}+\sqrt{5}=\dfrac{5}{2}\sqrt{5}$.

53. Let (a,b) be the coordinates of T (or T'). The radius from $(0,0)$ to T (or T') will be perpendicular to L (or L') so, using slopes, $b/a=-(a-3)/b$, $a^2+b^2=3a$. But (a,b) is on the circle so $a^2+b^2=1$, thus $3a=1$, $a=1/3$. Let $a=1/3$ in $a^2+b^2=1$ to get $b^2=8/9$, $b=\pm\sqrt{8}/3$. The coordinates of T and T' are $(1/3,\sqrt{8}/3)$ and $(1/3,-\sqrt{8}/3)$.

54. **(a)** $\sqrt{(x-2)^2+(y-0)^2}=\sqrt{2}\sqrt{(x-0)^2+(y-1)^2}$; square both sides and expand to get $x^2-4x+4+y^2=2(x^2+y^2-2y+1)$, $x^2+y^2+4x-4y-2=0$, which is a circle.

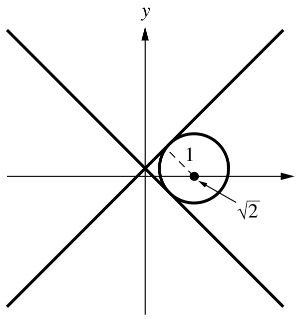
(b) $(x^2+4x)+(y^2-4y)=2$, $(x^2+4x+4)+(y^2-4y+4)=2+4+4$, $(x+2)^2+(y-2)^2=10$; center $(-2,2)$, radius $\sqrt{10}$.

55. **(a)** $[(x-4)^2+(y-1)^2]+[(x-2)^2+(y+5)^2]=45$
$x^2-8x+16+y^2-2y+1+x^2-4x+4+y^2+10y+25=45$
$2x^2+2y^2-12x+8y+1=0$, which is a circle.

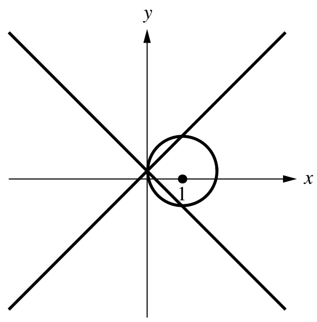
(b) $2(x^2-6x)+2(y^2+4y)=-1$, $2(x^2-6x+9)+2(y^2+4y+4)=-1+18+8$,
$(x-3)^2+(y+2)^2=25/2$; center $(3,-2)$, radius $5/\sqrt{2}$.

56. If $x^2-y^2=0$, then $y^2=x^2$ so $y=x$ or $y=-x$. The graph of $x^2-y^2=0$ consists of the graphs of the two lines $y=\pm x$. The graph of $(x-c)^2+y^2=1$ is a circle of radius 1 with center at $(c,0)$.

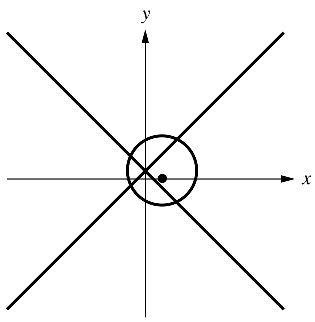
Examine the figure to see that the system cannot have just one solution, and has 0 solutions if $|c| > \sqrt{2}$, 2 solutions if $|c| = \sqrt{2}$, 3 solutions if $|c| = 1$, and 4 solutions if $|c| < \sqrt{2}$, $|c| \neq 1$.



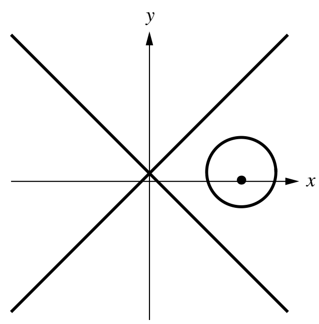
2 solutions



3 solutions

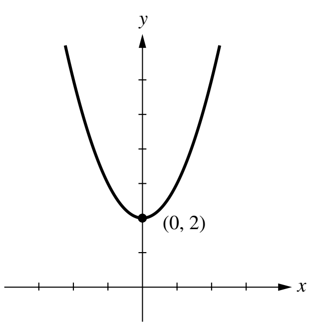


4 solutions

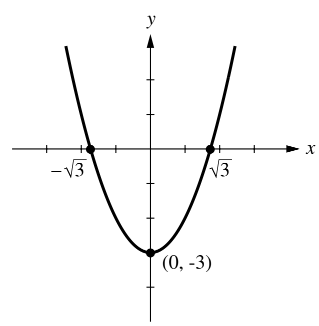


0 solutions

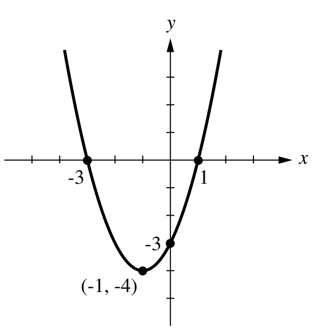
57. $y = x^2 + 2$



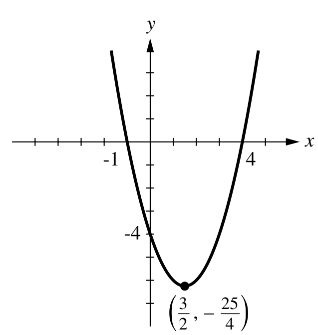
58. $y = x^2 - 3$



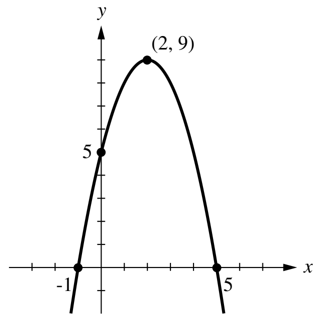
59. $y = x^2 + 2x - 3$



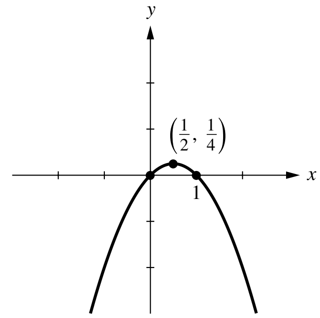
60. $y = x^2 - 3x - 4$



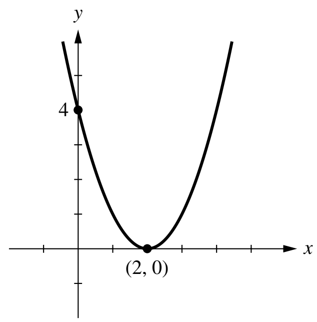
61. $y = -x^2 + 4x + 5$



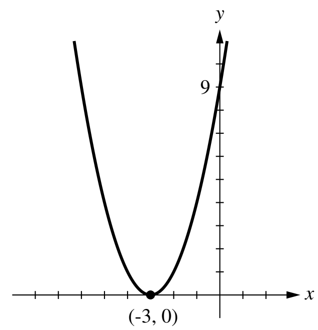
62. $y = -x^2 + x$



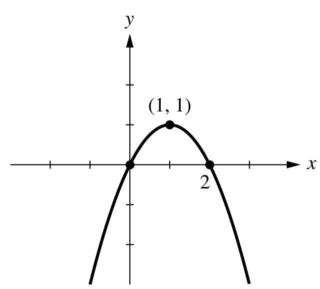
63. $y = (x-2)^2$



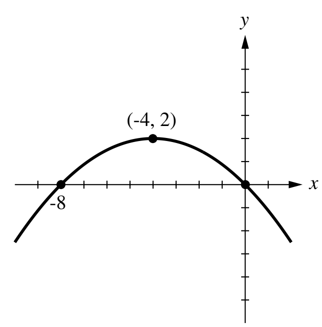
64. $y = (3+x)^2$



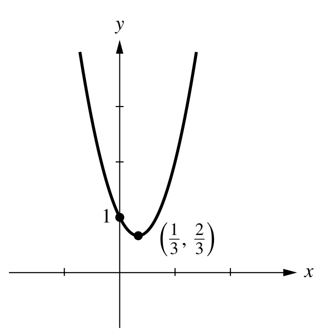
65. $x^2 - 2x + y = 0$



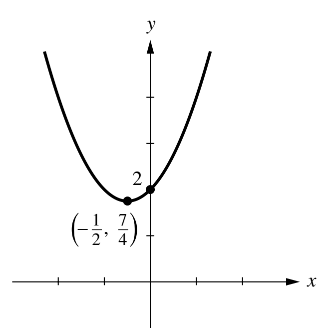
66. $x^2 + 8x + 8y = 0$



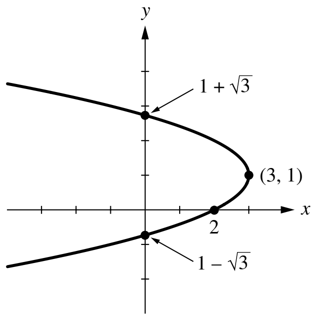
67. $y = 3x^2 - 2x + 1$



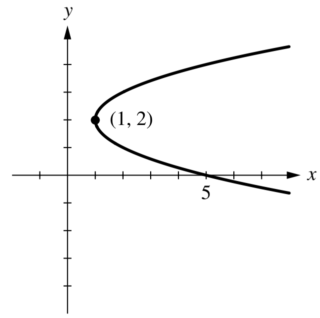
68. $y = x^2 + x + 2$



69. $x = -y^2 + 2y + 2$



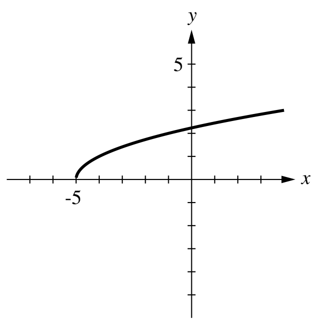
70. $x = y^2 - 4y + 5$



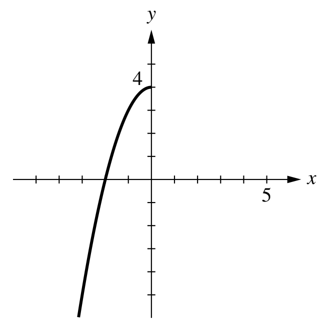
71. **(a)** $x^2 = 3 - y$, $x = \pm\sqrt{3-y}$. The right half is $x = \sqrt{3-y}$.
(b) Complete the square in x to get $(x-1)^2 = y+1$, $x = 1\pm\sqrt{y+1}$. The left half is $x = 1-\sqrt{y+1}$.

72. **(a)** $y^2 = x + 5$, $y = \pm\sqrt{x+5}$. The upper half is $y = \sqrt{x+5}$.
(b) Complete the square in y to get $(y - 1/2)^2 = x + 9/4$, $y - 1/2 = \pm\sqrt{x + 9/4}$, $y = 1/2 \pm \sqrt{x+9/4}$. The lower half is $y = 1/2 - \sqrt{x+9/4}$.

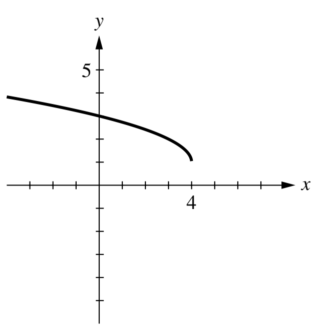
73. **(a)**



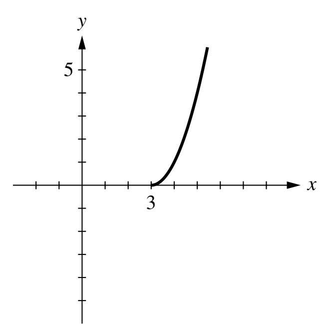
(b)



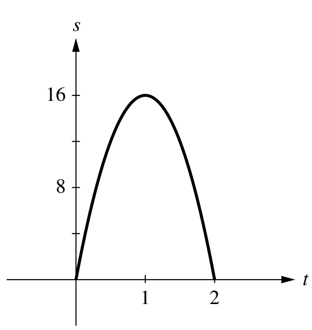
74. **(a)**



(b)



75. **(a)**



(b) The ball will be at its highest point when $t = 1$ sec; it will rise 16 ft.

76. **(a)** $2x + y = 500$, $y = 500 - 2x$. **(b)** $A = xy = x(500 - 2x) = 500x - 2x^2$.
(c) The graph of A versus x is a parabola with its vertex (high point) at $x = -b/(2a) = -500/(-4) = 125$, so the maximum value of A is $A = 500(125) - 2(125)^2 = 31{,}250$ ft^2.

77. **(a)** $(3)(2x) + (2)(2y) = 600$, $6x + 4y = 600$, $y = 150 - 3x/2$
(b) $A = xy = x(150 - 3x/2) = 150x - 3x^2/2$
(c) The graph of A versus x is a parabola with its vertex (high point) at $x = -b/(2a) = -150/(-3) = 50$, so the maximum value of A is $A = 150(50) - 3(50)^2/2 = 3{,}750$ ft^2.

78. **(a)** $y = ax^2 + bx + c = a\left(x^2 + \dfrac{b}{a}x\right) + c$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a} = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

(b) If $a < 0$ then y is always less than $c - \dfrac{b^2}{4a}$ except when $x = -\dfrac{b}{2a}$, so the graph has its high point there. If $a > 0$ then y is always greater than $c - \dfrac{b^2}{4a}$ except when $x = -\dfrac{b}{2a}$, so the graph has its low point there.

79. **(a)** The parabola $y = 2x^2 + 5x - 1$ opens upward and has x-intercepts of $x = (-5 \pm \sqrt{33})/4$, so $2x^2 + 5x - 1 < 0$ if $(-5 - \sqrt{33})/4 < x < (-5 + \sqrt{33})/4$.
(b) The parabola $y = x^2 - 2x + 3$ opens upward and has no x-intercepts, so $x^2 - 2x + 3 > 0$ if $-\infty < x < +\infty$.

80. **(a)** The parabola $y = x^2 + x - 1$ opens upward and has x-intercepts of $x = (-1 \pm \sqrt{5})/2$, so $x^2 + x - 1 > 0$ if $x < (-1 - \sqrt{5})/2$ or $x > (-1 + \sqrt{5})/2$.
(b) The parabola $y = x^2 - 4x + 6$ opens upward and has no x-intercepts, so $x^2 - 4x + 6 < 0$ has no solution.

81. **(a)** The t-coordinate of the vertex is $t = -40/[(2)(-16)] = 5/4$, so the maximum height is $s = 5 + 40(5/4) - 16(5/4)^2 = 30$ ft.
(b) $s = 5 + 40t - 16t^2 = 0$ if $t \approx 2.6$ s
(c) $s = 5 + 40t - 16t^2 > 12$ if $16t^2 - 40t + 7 < 0$, which is true if $(5 - 3\sqrt{2})/4 < t < (5 + 3\sqrt{2})/4$. The length of this interval is $(5 + 3\sqrt{2})/4 - (5 - 3\sqrt{2})/4 = 3\sqrt{2}/2 \approx 2.1$ s.

82. $x + 3 - x^2 > 0$, $x^2 - x - 3 < 0$, $(1 - \sqrt{13})/2 < x < (1 + \sqrt{13})/2$

APPENDIX E
Trigonometry Review

EXERCISE SET E

1. **(a)** $5\pi/12$ **(b)** $13\pi/6$ **(c)** $\pi/9$ **(d)** $23\pi/30$

2. **(a)** $7\pi/3$ **(b)** $\pi/12$ **(c)** $5\pi/4$ **(d)** $11\pi/12$

3. **(a)** $12°$ **(b)** $(270/\pi)°$ **(c)** $288°$ **(d)** $540°$

4. **(a)** $18°$ **(b)** $(360/\pi)°$ **(c)** $72°$ **(d)** $210°$

5.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$\sqrt{21}/5$	$2/5$	$\sqrt{21}/2$	$5/\sqrt{21}$	$5/2$	$2/\sqrt{21}$
(b)	$3/4$	$\sqrt{7}/4$	$3/\sqrt{7}$	$4/3$	$4/\sqrt{7}$	$\sqrt{7}/3$
(c)	$3/\sqrt{10}$	$1/\sqrt{10}$	3	$\sqrt{10}/3$	$\sqrt{10}$	$1/3$

6.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$1/\sqrt{2}$	$1/\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
(b)	$3/5$	$4/5$	$3/4$	$5/3$	$5/4$	$4/3$
(c)	$1/4$	$\sqrt{15}/4$	$1/\sqrt{15}$	4	$4/\sqrt{15}$	$\sqrt{15}$

7. $\sin\theta = 3/\sqrt{10}$, $\cos\theta = 1/\sqrt{10}$

8. $\sin\theta = \sqrt{5}/3$, $\tan\theta = \sqrt{5}/2$

9. $\tan\theta = \sqrt{21}/2$, $\csc\theta = 5/\sqrt{21}$

10. $\cot\theta = \sqrt{15}$, $\sec\theta = 4/\sqrt{15}$

11. Let x be the length of the side adjacent to θ, then $\cos\theta = x/6 = 0.3$, $x = 1.8$.

12. Let x be the length of the hypotenuse, then $\sin\theta = 2.4/x = 0.8$, $x = 2.4/0.8 = 3$.

13.

	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$225°$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
(b)	$-210°$	$1/2$	$-\sqrt{3}/2$	$-1/\sqrt{3}$	2	$-2/\sqrt{3}$	$-\sqrt{3}$
(c)	$5\pi/3$	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$	$-2/\sqrt{3}$	2	$-1/\sqrt{3}$
(d)	$-3\pi/2$	1	0	—	1	—	0

14.

	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$330°$	$-1/2$	$\sqrt{3}/2$	$-1/\sqrt{3}$	-2	$2/\sqrt{3}$	$-\sqrt{3}$
(b)	$-120°$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$	$-2/\sqrt{3}$	-2	$1/\sqrt{3}$
(c)	$9\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
(d)	-3π	0	-1	0	—	-1	—

15.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$4/5$	$3/5$	$4/3$	$5/4$	$5/3$	$3/4$
(b)	$-4/5$	$3/5$	$-4/3$	$-5/4$	$5/3$	$-3/4$
(c)	$1/2$	$-\sqrt{3}/2$	$-1/\sqrt{3}$	2	$-2\sqrt{3}$	$-\sqrt{3}$
(d)	$-1/2$	$\sqrt{3}/2$	$-1/\sqrt{3}$	-2	$2/\sqrt{3}$	$-\sqrt{3}$
(e)	$1/\sqrt{2}$	$1/\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
(f)	$1/\sqrt{2}$	$-1/\sqrt{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1

16.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$1/4$	$\sqrt{15}/4$	$1/\sqrt{15}$	4	$4/\sqrt{15}$	$\sqrt{15}$
(b)	$1/4$	$-\sqrt{15}/4$	$-1/\sqrt{15}$	4	$-4/\sqrt{15}$	$-\sqrt{15}$
(c)	$3/\sqrt{10}$	$1/\sqrt{10}$	3	$\sqrt{10}/3$	$\sqrt{10}$	$1/3$
(d)	$-3/\sqrt{10}$	$-1/\sqrt{10}$	3	$-\sqrt{10}/3$	$-\sqrt{10}$	$1/3$
(e)	$\sqrt{21}/5$	$-2/5$	$-\sqrt{21}/2$	$5/\sqrt{21}$	$-5/2$	$-2/\sqrt{21}$
(f)	$-\sqrt{21}/5$	$-2/5$	$\sqrt{21}/2$	$-5/\sqrt{21}$	$-5/2$	$2/\sqrt{21}$

17. **(a)** $x = 3\sin 25° \approx 1.2679$ **(b)** $x = 3/\tan(2\pi/9) \approx 3.5753$

18. **(a)** $x = 2/\sin 20° \approx 5.8476$ **(b)** $x = 3/\cos(3\pi/11) \approx 4.5811$

19.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$a/3$	$\sqrt{9-a^2}/3$	$a/\sqrt{9-a^2}$	$3/a$	$3/\sqrt{9-a^2}$	$\sqrt{9-a^2}/a$
(b)	$a/\sqrt{a^2+25}$	$5/\sqrt{a^2+25}$	$a/5$	$\sqrt{a^2+25}/a$	$\sqrt{a^2+25}/5$	$5/a$
(c)	$\sqrt{a^2-1}/a$	$1/a$	$\sqrt{a^2-1}$	$a/\sqrt{a^2-1}$	a	$1/\sqrt{a^2-1}$

20. **(a)** $\theta = 3\pi/4 \pm 2n\pi$ and $\theta = 5\pi/4 \pm 2n\pi$, $n = 0, 1, 2, \ldots$
(b) $\theta = 5\pi/4 \pm 2n\pi$ and $\theta = 7\pi/4 \pm 2n\pi$, $n = 0, 1, 2, \ldots$

21. **(a)** $\theta = 3\pi/4 \pm n\pi$, $n = 0, 1, 2, \ldots$
(b) $\theta = \pi/3 \pm 2n\pi$ and $\theta = 5\pi/3 \pm 2n\pi$, $n = 0, 1, 2, \ldots$

22. **(a)** $\theta = 7\pi/6 \pm 2n\pi$ and $\theta = 11\pi/6 \pm 2n\pi$, $n = 0, 1, 2, \ldots$
(b) $\theta = \pi/3 \pm n\pi$, $n = 0, 1, 2, \ldots$

23. **(a)** $\theta = \pi/6 \pm n\pi$, $n = 0, 1, 2, \ldots$
(b) $\theta = 4\pi/3 \pm 2n\pi$ and $\theta = 5\pi/3 \pm 2n\pi$, $n = 0, 1, 2, \ldots$

24. **(a)** $\theta = 3\pi/2 \pm 2n\pi$, $n = 0, 1, 2, \ldots$ **(b)** $\theta = \pi \pm 2n\pi$, $n = 0, 1, 2, \ldots$

25. **(a)** $\theta = 3\pi/4 \pm n\pi$, $n = 0, 1, 2, \ldots$ **(b)** $\theta = \pi/6 \pm n\pi$, $n = 0, 1, 2, \ldots$

26. **(a)** $\theta = 2\pi/3 \pm 2n\pi$ and $\theta = 4\pi/3 \pm 2n\pi$, $n = 0, 1, 2, \ldots$
(b) $\theta = 7\pi/6 \pm 2n\pi$ and $\theta = 11\pi/6 \pm 2n\pi$, $n = 0, 1, 2, \ldots$

27. **(a)** $\theta = \pi/3 \pm 2n\pi$ and $\theta = 2\pi/3 \pm 2n\pi$, $n = 0, 1, 2, \ldots$
(b) $\theta = \pi/6 \pm 2n\pi$ and $\theta = 11\pi/6 \pm 2n\pi$, $n = 0, 1, 2, \ldots$

28. $\sin\theta = -3/5$, $\cos\theta = -4/5$, $\tan\theta = 3/4$, $\csc\theta = -5/3$, $\sec\theta = -5/4$, $\cot\theta = 4/3$

29. $\sin\theta = 2/5$, $\cos\theta = -\sqrt{21}/5$, $\tan\theta = -2/\sqrt{21}$, $\csc\theta = 5/2$, $\sec\theta = -5/\sqrt{21}$, $\cot\theta = -\sqrt{21}/2$

30. **(a)** $\theta = \pi/2 \pm 2n\pi$, $n = 0, 1, 2, \ldots$ **(b)** $\theta = \pm 2n\pi$, $n = 0, 1, 2, \ldots$
(c) $\theta = \pi/4 \pm n\pi$, $n = 0, 1, 2, \ldots$ **(d)** $\theta = \pi/2 \pm 2n\pi$, $n = 0, 1, 2, \ldots$
(e) $\theta = \pm 2n\pi$, $n = 0, 1, 2, \ldots$ **(f)** $\theta = \pi/4 \pm n\pi$, $n = 0, 1, 2, \ldots$

31. **(a)** $\theta = \pm n\pi$, $n = 0, 1, 2, \ldots$ **(b)** $\theta = \pi/2 \pm n\pi$, $n = 0, 1, 2, \ldots$
(c) $\theta = \pm n\pi$, $n = 0, 1, 2, \ldots$ **(d)** $\theta = \pm n\pi$, $n = 0, 1, 2, \ldots$
(e) $\theta = \pi/2 \pm n\pi$, $n = 0, 1, 2, \ldots$ **(f)** $\theta = \pm n\pi$, $n = 0, 1, 2, \ldots$

32. Construct a right triangle with one angle equal to 17°, measure the lengths of the sides and hypotenuse and use formula (6) for $\sin\theta$ and $\cos\theta$ to approximate $\sin 17^\circ$ and $\cos 17^\circ$.

33. **(a)** $s = r\theta = 4(\pi/6) = 2\pi/3$ cm **(b)** $s = r\theta = 4(5\pi/6) = 10\pi/3$ cm

34. $r = s/\theta = 7/(\pi/3) = 21/\pi$ **35.** $\theta = s/r = 2/5$

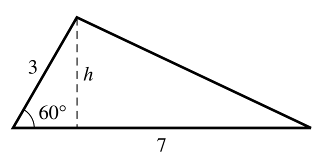
36. $\theta = s/r$ so $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}r^2(s/r) = \dfrac{1}{2}rs$

37. **(a)** $2\pi r = R(2\pi - \theta)$, $r = \dfrac{2\pi - \theta}{2\pi}R$

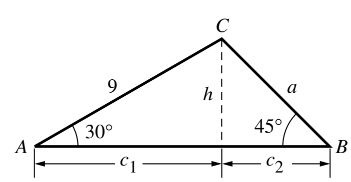
(b) $h = \sqrt{R^2 - r^2} = \sqrt{R^2 - (2\pi - \theta)^2 R^2/(4\pi^2)} = \dfrac{\sqrt{4\pi\theta - \theta^2}}{2\pi}R$

38. The circumference of the circular base is $2\pi r$. When cut and flattened, the cone becomes a circular sector of radius L. If θ is the central angle that subtends the arc of length $2\pi r$, then $\theta = (2\pi r)/L$ so the area S of the sector is $S = (1/2)L^2(2\pi r/L) = \pi r L$ which is the lateral surface area of the cone.

39. Let h be the altitude as shown in the figure, then
$h = 3\sin 60^\circ = 3\sqrt{3}/2$ so $A = \dfrac{1}{2}(3\sqrt{3}/2)(7) = 21\sqrt{3}/4$.



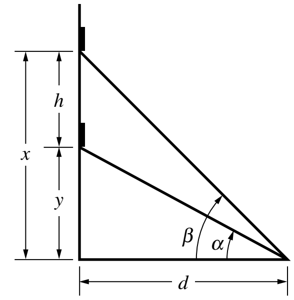
40. Draw the perpendicular from vertex C as shown in the figure, then
$h = 9\sin 30^\circ = 9/2$, $a = h/\sin 45^\circ = 9\sqrt{2}/2$,
$c_1 = 9\cos 30^\circ = 9\sqrt{3}/2$, $c_2 = a\cos 45^\circ = 9/2$,
$c_1 + c_2 = 9(\sqrt{3} + 1)/2$, angle $C = 180^\circ - (30^\circ + 45^\circ) = 105^\circ$



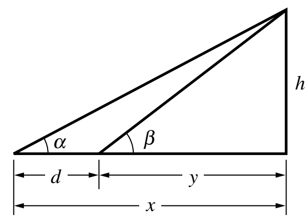
41. Let x be the distance above the ground, then $x = 10\sin 67^\circ \approx 9.2$ ft.

42. Let x be the height of the building, then $x = 120\tan 76^\circ \approx 481$ ft.

43. From the figure, $h = x - y$ but $x = d \tan \beta$, $y = d \tan \alpha$ so $h = d(\tan \beta - \tan \alpha)$.



44. From the figure, $d = x - y$ but $x = h \cot \alpha$, $y = h \cot \beta$ so $d = h(\cot \alpha - \cot \beta)$, $h = \dfrac{d}{\cot \alpha - \cot \beta}$.



45. **(a)** $\sin 2\theta = 2 \sin \theta \cos \theta = 2(\sqrt{5}/3)(2/3) = 4\sqrt{5}/9$
(b) $\cos 2\theta = 2\cos^2 \theta - 1 = 2(2/3)^2 - 1 = -1/9$

46. **(a)** $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta = (3/5)(1/\sqrt{5}) - (4/5)(2/\sqrt{5}) = -1/\sqrt{5}$
(b) $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = (4/5)(1/\sqrt{5}) - (3/5)(2/\sqrt{5}) = -2/(5\sqrt{5})$

47. $\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta = (2 \sin \theta \cos \theta) \cos \theta + (\cos^2 \theta - \sin^2 \theta) \sin \theta$ $= 2 \sin \theta \cos^2 \theta + \sin \theta \cos^2 \theta - \sin^3 \theta = 3 \sin \theta \cos^2 \theta - \sin^3 \theta$; similarly, $\cos 3\theta = \cos^3 \theta - 3 \sin^2 \theta \cos \theta$

48. $\dfrac{\cos \theta \sec \theta}{1 + \tan^2 \theta} = \dfrac{\cos \theta \sec \theta}{\sec^2 \theta} = \dfrac{\cos \theta}{\sec \theta} = \dfrac{\cos \theta}{(1/\cos \theta)} = \cos^2 \theta$

49. $\dfrac{\cos \theta \tan \theta + \sin \theta}{\tan \theta} = \dfrac{\cos \theta (\sin \theta / \cos \theta) + \sin \theta}{\sin \theta / \cos \theta} = 2 \cos \theta$

50. $2 \csc 2\theta = \dfrac{2}{\sin 2\theta} = \dfrac{2}{2 \sin \theta \cos \theta} = \left(\dfrac{1}{\sin \theta}\right)\left(\dfrac{1}{\cos \theta}\right) = \csc \theta \sec \theta$

51. $\tan \theta + \cot \theta = \dfrac{\sin \theta}{\cos \theta} + \dfrac{\cos \theta}{\sin \theta} = \dfrac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \dfrac{1}{\sin \theta \cos \theta} = \dfrac{2}{2 \sin \theta \cos \theta} = \dfrac{2}{\sin 2\theta} = 2 \csc 2\theta$

52. $\dfrac{\sin 2\theta}{\sin \theta} - \dfrac{\cos 2\theta}{\cos \theta} = \dfrac{\sin 2\theta \cos \theta - \cos 2\theta \sin \theta}{\sin \theta \cos \theta} = \dfrac{\sin \theta}{\sin \theta \cos \theta} = \sec \theta$

53. $\dfrac{\sin \theta + \cos 2\theta - 1}{\cos \theta - \sin 2\theta} = \dfrac{\sin \theta + (1 - 2\sin^2 \theta) - 1}{\cos \theta - 2 \sin \theta \cos \theta} = \dfrac{\sin \theta (1 - 2 \sin \theta)}{\cos \theta (1 - 2 \sin \theta)} = \tan \theta$

54. Using (47), $2 \sin 2\theta \cos \theta = 2(1/2)(\sin \theta + \sin 3\theta) = \sin \theta + \sin 3\theta$

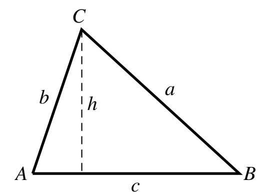
55. Using (47), $2 \cos 2\theta \sin \theta = 2(1/2)[\sin(-\theta) + \sin 3\theta] = \sin 3\theta - \sin \theta$

56. $\tan(\theta/2) = \dfrac{\sin(\theta/2)}{\cos(\theta/2)} = \dfrac{2 \sin^2(\theta/2)}{2 \sin(\theta/2) \cos(\theta/2)} = \dfrac{1 - \cos \theta}{\sin \theta}$

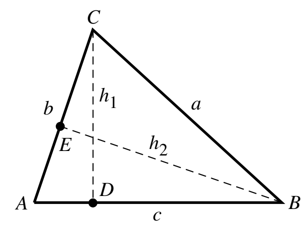
57. $\tan(\theta/2) = \dfrac{\sin(\theta/2)}{\cos(\theta/2)} = \dfrac{2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)} = \dfrac{\sin\theta}{1+\cos\theta}$

58. From (52), $\cos(\pi/3+\theta)+\cos(\pi/3-\theta) = 2\cos(\pi/3)\cos\theta = 2(1/2)\cos\theta = \cos\theta$

59. From the figures, area $= \dfrac{1}{2}hc$ but $h = b\sin A$ so area $= \dfrac{1}{2}bc\sin A$. The formulas area $= \dfrac{1}{2}ac\sin B$ and area $= \dfrac{1}{2}ab\sin C$ follow by drawing altitudes from vertices B and C, respectively.



60. From right triangles ADC and BDC,
$h_1 = b\sin A = a\sin B$ so $a/\sin A = b/\sin B$.
From right triangles AEB and CEB,
$h_2 = c\sin A = a\sin C$ so $a/\sin A = c/\sin C$
thus $a/\sin A = b/\sin B = c/\sin C$.



61. **(a)** $\sin(\pi/2+\theta) = \sin(\pi/2)\cos\theta + \cos(\pi/2)\sin\theta = (1)\cos\theta + (0)\sin\theta = \cos\theta$

(b) $\cos(\pi/2+\theta) = \cos(\pi/2)\cos\theta - \sin(\pi/2)\sin\theta = (0)\cos\theta - (1)\sin\theta = -\sin\theta$

(c) $\sin(3\pi/2-\theta) = \sin(3\pi/2)\cos\theta - \cos(3\pi/2)\sin\theta = (-1)\cos\theta - (0)\sin\theta = -\cos\theta$

(d) $\cos(3\pi/2+\theta) = \cos(3\pi/2)\cos\theta - \sin(3\pi/2)\sin\theta = (0)\cos\theta - (-1)\sin\theta = \sin\theta$

62. $\tan(\alpha+\beta) = \dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$, divide numerator and denominator by $\cos\alpha\cos\beta$ and use $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ and $\tan\beta = \dfrac{\sin\beta}{\cos\beta}$ to get (38);

$\tan(\alpha-\beta) = \tan(\alpha+(-\beta)) = \dfrac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)} = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$ because $\tan(-\beta) = -\tan\beta$.

63. **(a)** Add (34) and (36) to get $\sin(\alpha-\beta)+\sin(\alpha+\beta) = 2\sin\alpha\cos\beta$ so $\sin\alpha\cos\beta = (1/2)[\sin(\alpha-\beta)+\sin(\alpha+\beta)]$.

(b) Subtract (35) from (37).

(c) Add (35) and (37).

64. **(a)** From (47), $\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2} = \dfrac{1}{2}(\sin B+\sin A)$ so

$\sin A+\sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$.

(b) Use (49)

(c) Use (48)

65. $\sin\alpha+\sin(-\beta) = 2\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$, but $\sin(-\beta) = -\sin\beta$ so

$\sin\alpha\ -\sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$.

66. **(a)** From (34), $C\sin(\alpha+\phi) = C\sin\alpha\cos\phi + C\cos\alpha\sin\phi$ so $C\cos\phi = 3$ and $C\sin\phi = 5$, square and add to get $C^2(\cos^2\phi + \sin^2\phi) = 9 + 25$, $C^2 = 34$. If $C = \sqrt{34}$ then $\cos\phi = 3/\sqrt{34}$ and $\sin\phi = 5/\sqrt{34}$ so ϕ is the first-quadrant angle for which $\tan\phi = 5/3$.
$3\sin\alpha + 5\cos\alpha = \sqrt{34}\sin(\alpha+\phi)$.

(b) Follow the procedure of part (a) to get $C\cos\phi = A$ and $C\sin\phi = B$, $C = \sqrt{A^2+B^2}$, $\tan\phi = B/A$ where the quadrant in which ϕ lies is determined by the signs of A and B because $\cos\phi = A/C$ and $\sin\phi = B/C$, so $A\sin\alpha + B\cos\alpha = \sqrt{A^2+B^2}\sin(\alpha+\phi)$.

67. Consider the triangle having a, b, and d as sides. The angle formed by sides a and b is $\pi - \theta$ so from the law of cosines, $d^2 = a^2 + b^2 - 2ab\cos(\pi-\theta) = a^2 + b^2 + 2ab\cos\theta$, $d = \sqrt{a^2+b^2+2ab\cos\theta}$.

APPENDIX F
Solving Polynomial Equations

EXERCISE SET F

1. **(a)** $q(x) = x^2 + 4x + 2, r(x) = -11x + 6$
(b) $q(x) = 2x^2 + 4, r(x) = 9$
(c) $q(x) = x^3 - x^2 + 2x - 2, r(x) = 2x + 1$

2. **(a)** $q(x) = 2x^2 - x + 2, r(x) = 5x + 5$
(b) $q(x) = x^3 + 3x^2 - x + 2, r(x) = 3x - 1$
(c) $q(x) = 5x^3 - 5, r(x) = 4x^2 + 10$

3. **(a)** $q(x) = 3x^2 + 6x + 8, r(x) = 15$
(b) $q(x) = x^3 - 5x^2 + 20x - 100, r(x) = 504$
(c) $q(x) = x^4 + x^3 + x^2 + x + 1, r(x) = 0$

4. **(a)** $q(x) = 2x^2 + x - 1, r(x) = 0$
(b) $q(x) = 2x^3 - 5x^2 + 3x - 39, r(x) = 147$
(c) $q(x) = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1, r(x) = 2$

5.

x	0	1	-3	7
$p(x)$	-4	-3	101	5001

6.

x	1	-1	3	-3	7	-7	21	-21
$p(x)$	-24	-12	12	0	420	-168	10416	-7812

7. **(a)** $q(x) = x^2 + 6x + 13, r = 20$ **(b)** $q(x) = x^2 + 3x - 2, r = -4$

8. **(a)** $q(x) = x^4 - x^3 + x^2 - x + 1, r = -2$ **(b)** $q(x) = x^4 + x^3 + x^2 + x + 1, r = 0$

9. Assume $r = a/b$ a and b integers with $a > 0$:
(a) b divides 1, $b = \pm 1$; a divides 24, $a = 1, 2, 3, 4, 6, 8, 12, 24$;
the possible candidates are $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24\}$
(b) b divides 3 so $b = \pm 1, \pm 3$; a divides -10 so $a = 1, 2, 5, 10$;
the possible candidates are $\{\pm 1, \pm 2, \pm 5, \pm 10, \pm 1/3, \pm 2/3, \pm 5/3, \pm 10/3\}$
(c) b divides 1 so $b = \pm 1$; a divides 17 so $a = 1, 17$;
the possible candidates are $\{\pm 1, \pm 17\}$

10. An integer zero c divides -21, so $c = \pm 1, \pm 3, \pm 7, \pm 21$ are the only possibilities; substitution of these candidates shows that the integer zeros are $-7, -1, 3$

11. $(x + 1)(x - 1)(x - 2)$

12. $(x + 2)(3x + 1)(x - 2)$

13. $(x + 3)^3(x + 1)$

14. $2x^4 + x^3 - 19x^2 + 9$

15. $(x + 3)(x + 2)(x + 1)^2(x - 3)$

17. -3 is the only real root.

18. $x = -3/2, 2 \pm \sqrt{3}$ are the real roots.

19. $x = -2, -2/3, -1 \pm \sqrt{3}$ are the real roots.

20. $-2, -1, 1/2, 3$

21. $-2, 2, 3$ are the only real roots.

23. If $x - 1$ is a factor then $p(1) = 0$, so $k^2 - 7k + 10 = 0$, $k^2 - 7k + 10 = (k-2)(k-5)$, so $k = 2, 5$.

24. $(-3)^7 = -2187$, so -3 is a root and thus by Theorem F.4, $x + 3$ is a factor of $x^7 + 2187$.

25. If the side of the cube is x then $x^2(x-3) = 196$; the only real root of this equation is $x = 7$ cm.

26. **(a)** Try to solve $\frac{a}{b} > \left(\frac{a}{b}\right)^3 + 1$. The polynomial $p(x) = x^3 - x + 1$ has only one real root $c \approx -1.325$, and $p(0) = 1$ so $p(x) > 0$ for all $x > c$; hence there is no positive rational solution of $\frac{a}{b} > \left(\frac{a}{b}\right)^3 + 1$.

(b) From part (a), any real $x < c$ is a solution.

27. Use the Factor Theorem with x as the variable and y as the constant c.

(a) For any positive integer n the polynomial $x^n - y^n$ has $x = y$ as a root.

(b) For any positive even integer n the polynomial $x^n - y^n$ has $x = -y$ as a root.

(c) For any positive odd integer n the polynomial $x^n + y^n$ has $x = -y$ as a root.