1

a) $\lim n(1+(-1)^n) = \begin{cases} n(1+1) & \text{se } n \text{ par} \\ n(1-1) & \text{se } n \text{ ímpar} \end{cases} = \begin{cases} 2n & \text{se } n \text{ par} \\ 0 & \text{se } n \text{ ímpar} \end{cases}$

logo, $n(1+(-1)^n)$ não tem limite

b) $\lim (n^4-(-1)^n n^2) = \begin{cases} n^4 - n^2 & \text{se } n \text{ par} \\ n^4 + n^2 & \text{se } n \text{ ímpar} \end{cases}$

$= \begin{cases} n^2(n^2-1) & \text{se } n \text{ par} \\ n^2(n^2+1) & \text{se } n \text{ ímpar} \end{cases}$ ambos tendem para $+\infty$ pois n^4 cresce mais rápido que n^2

logo, $(n^4-(-1)^n n^2)$ tende para $+\infty$

c) $\lim \left(\log(n^2+1) - e^{n+1} + n^{100}\right) \to -\infty$, pois e^{n+1} cresce mais rápido que o logaritmo e que n^{100}

logo, $\left(\log(n^2+1) - e^{n+1} + n^{100}\right)$ tende para $-\infty$

2

a) $\lim \frac{1-n}{5n+3} = \lim \frac{\cancel{n}(\frac{1}{n}-1)}{\cancel{n}(5+\frac{3}{n})} = \frac{0^+-1}{5+0^+} = -\frac{1}{5}$

logo, $\frac{1-n}{5n+3}$ tende para $-\frac{1}{5}$

b) $\lim \frac{n^2+2}{3n+1} = \lim \frac{\cancel{n^2}(1+\frac{2}{n^2})}{\cancel{n^2}(\frac{3}{n}+\frac{1}{n^2})} = \frac{1+0^+}{0^++0^+} = \frac{1}{0^+} = +\infty$

logo, $\frac{n^2+2}{3n+1}$ tende para $+\infty$

c) $\lim \frac{3n}{4n^3+1} = \lim \frac{\cancel{n^3}(\frac{3}{n^2})}{\cancel{n^3}(4+\frac{1}{n^3})} = \frac{0^+}{4+0^+} = \frac{0^+}{4} = 0$

logo $\frac{3n}{4n^3+1}$ tende para 0

d) $\lim \frac{-n^3+2}{4n^3-7} = \lim \frac{\cancel{n^3}(-1+\frac{2}{n^3})}{\cancel{n^3}(4-\frac{7}{n^3})} = \frac{-1+0^+}{4-0^+} = -\frac{1}{4}$

logo $\frac{-n^3+2}{4n^3-7}$ tende para $-\frac{1}{4}$

③

a) $\lim \dfrac{\sqrt{n+1}\,(1+2\sqrt{n})}{n+\sqrt[3]{n}} = \lim \dfrac{\sqrt{n+1}+2\sqrt{n^2+n}}{n+\sqrt[3]{n}} =$

$= \lim \dfrac{\frac{\sqrt{n+1}}{n}+\frac{2\sqrt{n^2+n}}{n}}{\frac{n+\sqrt[3]{n}}{n}} = \lim \dfrac{\sqrt{\frac{n+1}{n^2}}+2\sqrt{\frac{n^2}{n^2}+\frac{n}{n^2}}}{1+\sqrt[3]{\frac{n}{n^3}}} =$

$= \lim \dfrac{\sqrt{\frac{1}{n}+\frac{1}{n^2}}+2\sqrt{1+\frac{1}{n}}}{1+\sqrt[3]{\frac{1}{n^2}}} = \dfrac{\sqrt{0^+ + 0^+}+2\sqrt{1+0^+}}{1+\sqrt[3]{0^+}} = \dfrac{2}{1} = 2$

Logo, $\dfrac{\sqrt{n+1}\,(1+2\sqrt{n})}{n+\sqrt[3]{n}}$ tende para 2

b) $\lim \dfrac{n((-1)^n+\sqrt{n})}{2+\sqrt{n^3+1}} = \lim \dfrac{n(-1)^n+n\sqrt{n}}{2+\sqrt{n^3+1}} =$

$= \lim \dfrac{n(-1)^n+\sqrt{n^3}}{2+\sqrt{n^3+1}} = \lim \dfrac{\frac{n(-1)^n}{\sqrt{n^3}}+\frac{\sqrt{n^3}}{\sqrt{n^3}}}{\frac{2}{\sqrt{n^3}}+\frac{\sqrt{n^3+1}}{\sqrt{n^3}}} =$

$= \lim \dfrac{\frac{\cancel{n}(-1)^n}{\cancel{n}\sqrt{n}}+1}{\frac{2}{\sqrt{n^3}}+\sqrt{1+\frac{1}{n^3}}} = \dfrac{0^+ + 1}{0^+ + 1} = \dfrac{1}{1} = 1$

Logo, $\dfrac{n((-1)^n+\sqrt{n})}{2+\sqrt{n^3+1}}$ tende para 1

c) $\lim \dfrac{n\sqrt[3]{n^2+2}}{n^2+(-1)^n n} = \lim \dfrac{\sqrt[3]{n^3(n^2+2)}}{n^2+(-1)^n n} =$

$= \lim \dfrac{\sqrt[3]{n^5+2n^3}}{n^2+(-1)^n n} = \lim \dfrac{\sqrt[3]{\frac{n^5+2n^3}{n^5}}}{\frac{n^2}{n^{\frac{5}{3}}}+(-1)^n\frac{n}{n^{\frac{5}{3}}}} =$

$= \lim \dfrac{\sqrt[3]{1+\frac{2}{n^2}}}{\sqrt[3]{n}+\frac{(-1)^n}{\sqrt[3]{n^2}}} = \dfrac{1+0^+}{+\infty+0^+} = \dfrac{1}{+\infty} = 0$

Logo, $\dfrac{n\sqrt[3]{n^2+2}}{n^2+(-1)^n n}$ Tende para 0

d) $\lim \dfrac{2n\,e^{\frac{1}{n}}}{(-1)^n+\sqrt{n^2+5}} = \lim \dfrac{n(2e^{\frac{1}{n}})}{(-1)^n+\sqrt{n^2(1+\frac{5}{n^2})}} =$

$= \lim \dfrac{\cancel{n}(2e^{\frac{1}{n}})}{\cancel{n}\left(\frac{(-1)^n}{n}+\sqrt{1+\frac{5}{n^2}}\right)} = \dfrac{2\times 1}{0^+ + 1} = \dfrac{2}{1} = 2$

Logo, $\dfrac{2n\,e^{\frac{1}{n}}}{(-1)^n+\sqrt{n^2+5}}$ tende para 2

4)

a) $\lim \frac{4^n}{2^{2n+1}+2^n} = \lim \frac{2^{2n}}{2^{2n+1}+2^n} =$

$= \lim \frac{\cancel{2^{2n}}}{\cancel{2^{2n}}(2^1+2^{-n})} = \lim \frac{1}{2+\frac{1}{2^n}} = \frac{1}{2+0^+} = \frac{1}{2}$

Logo, $\frac{4^n}{2^{2n+1}+2^n}$ tende para $\frac{1}{2}$

b) $\lim \frac{2^{3n}+5^n}{8^n+1} = \lim \frac{8^n+5^n}{8^n+1} =$

$= \lim \frac{\cancel{8^n}\left(1+\left(\frac{5}{8}\right)^n\right)}{\cancel{8^n}\left(1+\left(\frac{1}{8}\right)^n\right)} = \frac{1+0^+}{1+0^+} = \frac{1}{1} = 1$

Logo, $\frac{2^{3n}+5^n}{8^n+1}$ tende para 1

c) $\lim \frac{3^n}{2^{2n+1}+2^n} = \lim \frac{\cancel{3^n}}{\cancel{3^n}\left(\frac{2^{2n+1}}{3^n}+\frac{2^n}{3^n}\right)} =$

$= \lim \frac{1}{2\times\left(\frac{4}{3}\right)^n+\left(\frac{2}{3}\right)^n} = \frac{1}{2\times+\infty+0^+} = \frac{1}{+\infty} = 0$

Logo, $\frac{3^n}{2^{2n+1}+2^n}$ tende para 0

d) $\lim \frac{2^n+3^{n+1}+(-7)^n}{3^{n-2}+5^{n+1}} = \lim \frac{\cancel{(-7)^n}\left(-\left(\frac{2}{7}\right)^n-\frac{3^{n+1}}{7^n}+1\right)}{\cancel{(-7)^n}\left(-\frac{3^{n-2}}{7^n}-\frac{5^{n+1}}{7^n}\right)} =$

$= \lim \frac{-\left(\frac{2}{7}\right)^n-3\times\left(\frac{3}{7}\right)^n+1}{-\frac{1}{9}\times\left(\frac{3}{7}\right)^n-5\times\left(\frac{5}{7}\right)^n} = \frac{-0^+-3\times0^++1}{-\frac{1}{9}\times0^+-5\times0^+} = \frac{1}{0^+} = +\infty$

Logo, $\frac{2^n+3^{n+1}+(-7)^n}{3^{n-2}+5^{n+1}}$ tende para $+\infty$

5)

a) $\lim\left(\sqrt{n+1}-\sqrt{n}\right) = \lim \frac{\left(\sqrt{n+1}-\sqrt{n}\right)\left(\sqrt{n+1}+\sqrt{n}\right)}{\sqrt{n+1}+\sqrt{n}} =$

$= \lim \frac{\cancel{n}+1-\cancel{n}}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{+\infty+\infty} = \frac{1}{+\infty} = 0$

Logo, $\sqrt{n+1}-\sqrt{n}$ tende para 0

b) $\lim \dfrac{2n+1}{n(\sqrt{n+2}-\sqrt{n+1})} = \lim \dfrac{\cancel{n}(2+\frac{1}{n})}{\cancel{n}(\sqrt{n+2}-\sqrt{n+1})} =$

$= \lim \dfrac{2+\frac{1}{n}}{\sqrt{n+2}-\sqrt{n+1}} = \lim \dfrac{(2+\frac{1}{n})(\sqrt{n+2}+\sqrt{n+1})}{(\sqrt{n+2}-\sqrt{n+1})(\sqrt{n+2}+\sqrt{n+1})} =$

$= \lim \dfrac{(2+\frac{1}{n})(\sqrt{n+2}+\sqrt{n+1})}{\cancel{n}+2-\cancel{n}+1} = \dfrac{2 \times +\infty}{3} = +\infty$

Logo, $\dfrac{2n+1}{n(\sqrt{n+2}-\sqrt{n+1})}$ tende para $+\infty$

c) $\lim \left(\sqrt{\log(n^2+1)} - \sqrt{\log(n^2)}\right) =$

$= \lim \frac{1}{2}\log(n^2+1) - \frac{1}{2}\log(n^2) =$

$= \lim \frac{1}{2}\log\left(\frac{n^2+1}{n^2}\right) =$

$= \lim \frac{1}{2}\log\left(1+\frac{1}{n^2}\right) = \frac{1}{2}\log(1) = \frac{1}{2} \times 0 = 0$

Logo, $\sqrt{\log(n^2+1)} - \sqrt{\log(n^2)}$ tende para 0

6.

a) $\lim \left(\dfrac{n+3}{n+1}\right)^{2n} \underset{(i)}{=} \lim \left(1+\dfrac{2}{n+1}\right)^{2n} =$

$$\begin{array}{r|l} (i)\ n+3 & n+1 \\ -n-1 & 1 \\ \hline 0+2 & \end{array}$$

$= \lim \left(1+\dfrac{2}{n+1}\right)^{2n+2-2} = \dfrac{\lim \left(1+\frac{2}{n+1}\right)^{2n+2}}{\lim \left(1+\frac{2}{n+1}\right)^{2}} =$

$= \dfrac{\left(\lim\left(1+\frac{2}{n+1}\right)^{n+1}\right)^2}{1} = (e^2)^2 = e^4$

Logo, $\left(\dfrac{n+3}{n+1}\right)^{2n}$ tende para e^4

b) $\lim \left(\dfrac{n+5}{2n+1}\right)^{n} = \lim\left(\lim\left(\dfrac{n+5}{2n+1}\right)\right)^n =$

$= \lim\left(\lim\left(\dfrac{\cancel{n}(1+\frac{5}{n})}{\cancel{n}(2+\frac{1}{n})}\right)\right)^n = \lim\left(\dfrac{1+0^+}{2+0^+}\right)^n =$

$= \lim\left(\frac{1}{2}\right)^n = 0$

Logo, $\left(\dfrac{n+5}{2n+1}\right)^{n}$ tende para 0

c) $\lim\left(1-\frac{3}{n^2}\right)^n = \lim \sqrt[n]{\left(1-\frac{3}{n^2}\right)^{n^2}} = 1$

logo, $\left(1-\frac{3}{n^2}\right)^n$ tende para 1

d) $\lim n \log\left(1+\frac{5}{n^5}\right) = \lim \log\left(1+\frac{5}{n^5}\right)^n =$

$= \log(1)^{+\infty} = 0$

logo, $n \log\left(1+\frac{5}{n^5}\right)$ tende para 0

e) $\lim\left(1+\frac{1}{\log(n)}\right)^n = \lim\left(1+\log(n)^{-1}\right)^n =$

$= \lim\left(1-\log(n)\right)^n = +\infty$

logo, $\left(1+\frac{1}{\log(n)}\right)^n$ tende para $+\infty$

f) $\lim\left(\left(\frac{2n-1}{2n+1}\right)^n + \left(\frac{3}{5}\right)^n\right) \underset{(i)}{=}$

$$\begin{array}{r|l} (i)\; 2n-1 & 2n+1 \\ -2n-1 & 1 \\ \hline 0\;-2 & \end{array}$$

$= \lim\left(\left(1+\frac{-2}{2n+1}\right)^n + \left(\frac{3}{5}\right)^n\right) =$

$= \lim\left(\left(1+\frac{-1}{n+\frac{1}{2}}\right)^n + \left(\frac{3}{5}\right)^n\right) = \lim\left(\left(1+\frac{-1}{n+\frac{1}{2}}\right)^{n+\frac{1}{2}-\frac{1}{2}} + \left(\frac{3}{5}\right)^n\right) =$

$= \lim\left(\frac{\left(1+\frac{-1}{n+\frac{1}{2}}\right)^{n+\frac{1}{2}}}{\left(1+\frac{-1}{n+\frac{1}{2}}\right)^{\frac{1}{2}}} + \left(\frac{3}{5}\right)^n\right) = \frac{e^{-1}}{1} + 0^+ = \frac{1}{e}$

logo, $\left(\frac{2n-1}{2n+1}\right)^n + \left(\frac{3}{5}\right)^n$ tende para $\frac{1}{e}$

g) $\lim\left(\frac{n^6-2}{n^6}\right)^{n^3+3} = \lim\left(1-\frac{2}{n^6}\right)^{n^3+3} =$

$= \lim\left(1+\frac{-2}{n^6}\right)^{n^3} \times \lim\left(1+\frac{-2}{n^6}\right)^3 =$

$= \lim \sqrt[n^3]{\underbrace{\left(1+\frac{-2}{n^6}\right)^{n^6}}_{e^{-2}}} \times 1 = 1$

logo, $\left(\frac{n^6-2}{n^6}\right)^{n^3+3}$ tende para 1

h) $\lim \left(\frac{3n-3}{3n+1}\right)^{\frac{n+1}{3}} \underset{(i)}{=} \lim \left(1+\frac{-4}{3n+1}\right)^{\frac{n+1}{3}} =$

(i) $\begin{array}{r|l} 3n-3 & 3n+1 \\ -3n-1 & 1 \\ \hline 0-4 & \end{array}$

$= \left(\lim \left(1+\frac{-4}{3n+1}\right)^{n+1}\right)^{\frac{1}{3}} = \left(\lim \left(1+\frac{-4}{3n+1}\right)^{3n+1-2n}\right)^{\frac{1}{3}} =$

$= \left(\lim \frac{\left(1+\frac{-4}{3n+1}\right)^{3n+1}}{\left(1+\frac{-4}{3n+1}\right)^{2n}}\right)^{\frac{1}{3}} = \left(\lim \frac{\left(1+\frac{-4}{3n+1}\right)^{3n+1}}{\left(\left(1+\frac{-4}{3n+1}\right)^{3n+1}\right)^{\frac{2n}{3n+1}}}\right)^{\frac{1}{3}} =$

$= \left(\lim \frac{\left(1+\frac{-4}{3n+1}\right)^{3n+1}}{\left(\left(1+\frac{-4}{3n+1}\right)^{3n+1}\right)^{\frac{\cancel{n}(2)}{\cancel{n}(3+\frac{1}{n})}}}\right)^{\frac{1}{3}} = \left(\frac{e^{-4}}{(e^{-4})^{\frac{2}{3}}}\right)^{\frac{1}{3}} = \left(\frac{e^{-4}}{e^{-\frac{8}{3}}}\right)^{\frac{1}{3}} =$

$= \left(e^{-\frac{4}{3}}\right)^{\frac{1}{3}} = e^{-\frac{4}{9}} = \frac{1}{\sqrt[9]{e^4}}$

Logo, $\left(\frac{3n-3}{3n+1}\right)^{\frac{n+1}{3}}$ tende para $\frac{1}{\sqrt[9]{e^4}}$

7

a) $\lim \sum_{i=1}^{n+1} \frac{3}{\sqrt{9n^2+2i}}$ Seja $w_n = \sum_{i=1}^{n+1} \frac{3}{\sqrt{9n^2+2i}}$

Sendo $u_n \leq w_n \leq v_n$, se $\lim u_n = \lim v_n$, então, $\lim w_n$ será igual a $\lim u_n$

$u_n = \frac{3}{\sqrt{9n^2+2(n+1)}} \times (n+1) = \frac{3n+3}{\sqrt{9n^2+2n+2}} = \frac{\cancel{n}(3+\frac{3}{n})}{\cancel{n}\sqrt{9+\frac{2}{n}+\frac{2}{n^2}}} \longrightarrow \frac{3+0^+}{\sqrt{9+0^++0^+}} = \frac{3}{\sqrt{9}} = \frac{3}{3} = 1$

$v_n = \frac{3}{\sqrt{9n^2+1}} \times (n+1) = \frac{3n+3}{\sqrt{9n^2+1}} = \frac{\cancel{n}(3+\frac{3}{n})}{\cancel{n}\sqrt{9+\frac{1}{n}}} \longrightarrow \frac{3+0^+}{\sqrt{9+0^+}} = \frac{3}{\sqrt{9}} = \frac{3}{3} = 1$

Logo, w_n tende para 1

b) $\lim \sum_{k=0}^{3n} \frac{n}{\sqrt{4n^2+k}}$ Seja $w_n = \sum_{k=0}^{3n} \frac{n}{\sqrt{4n^2+k}}$

Sendo $u_n \leq w_n \leq v_n$, se $\lim u_n = \lim v_n$, então, $\lim u_n$ será igual a $\lim u_n$

$u_n = \frac{n}{\sqrt{4n^2+3n}} \times (3n+1) = \frac{3n^2+n}{\sqrt{4n^2+3n}} = \frac{\cancel{n^2}(3+\frac{1}{n})}{\cancel{n^2}\sqrt{\frac{4}{n^2}+\frac{3}{n^3}}} \longrightarrow \frac{3+0^+}{\sqrt{0^++0^+}} = \frac{3}{0^+} = +\infty$

$v_n = \frac{n}{\sqrt{4n^2}} \times (3n+1) = \frac{3n^2+n}{2n} = \frac{\cancel{n^2}(3+\frac{1}{n})}{\cancel{n^2}(\frac{2}{n})} \longrightarrow \frac{3+0^+}{0^+} = \frac{3}{0^+} = +\infty$

Logo, w_n tende para $+\infty$

c) $\lim \sum_{k=1}^{2n+3} \frac{2n+1}{\sqrt{n^4+k}}$ Seja $w_n = \sum_{k=1}^{2n+3} \frac{2n+1}{\sqrt{n^4+k}}$

Sendo $u_n \leq w_n \leq v_n$, se $\lim u_n = \lim v_n$, então, $\lim w_n$ será igual a $\lim u_n$

$$u_n = \frac{2n+1}{\sqrt{n^4+2n+3}} \times (2n+3) = \frac{4n^2+6n+2n+3}{\sqrt{n^4+2n+3}} = \frac{4n^2+8n+3}{\sqrt{n^4+2n+3}} =$$

$$= \frac{\cancel{n^2}\left(4+\frac{8}{n}+\frac{3}{n^2}\right)}{\cancel{n^2}\sqrt{1+\frac{2}{n^3}+\frac{3}{n^4}}} \rightarrow \frac{4+0^++0^+}{\sqrt{1+0^++0^+}} = \frac{4}{\sqrt{1}} = \frac{4}{1} = 4$$

$$v_n = \frac{2n+1}{\sqrt{n^4+5}} \times (2n+3) = \frac{4n^2+8n+3}{\sqrt{n^4+5}} = \frac{\cancel{n^2}\left(4+\frac{8}{n}+\frac{3}{n^2}\right)}{\cancel{n^2}\sqrt{1+\frac{5}{n^4}}} \rightarrow \frac{4+0^++0^+}{\sqrt{1+0^+}} =$$

$$= \frac{4}{\sqrt{1}} = \frac{4}{1} = 4$$

Logo, w_n tende para 4

d) $\lim \sum_{k=3}^{n^2+1} \frac{\operatorname{sen}(n^2+1)}{n^3-2k}$ Seja $w_n = \sum_{k=3}^{n^2+1} \frac{\operatorname{sen}(n^2+1)}{n^3-2k}$

Sendo $u_n \leq w_n \leq v_n$, se $\lim u_n = \lim v_n$, então, $\lim w_n$ será igual a $\lim u_n$

$$u_n = \frac{\operatorname{sen}(n^2+1)}{n^3-2(n^2+1)} \times (n^2-1) = \frac{\operatorname{sen}(n^2+1)(n^2-1)}{n^3-2n^2-2} =$$

$$= \frac{\cancel{n^3}\left(\frac{1}{n}-\frac{1}{n^3}\right)\operatorname{sen}(n^2+1)}{\cancel{n^3}\left(1-\frac{2}{n}-\frac{2}{n^2}\right)} \rightarrow \frac{0^+}{1} = 0$$

$$v_n = \frac{\operatorname{sen}(n^2+1)}{n^3-2\times 10} \times (n^2-1) = \frac{(n^2-1)\operatorname{sen}(n^2+1)}{n^3-20} =$$

$$= \frac{\cancel{n^3}\left(\frac{1}{n}-\frac{1}{n^3}\right)\operatorname{sen}(n^2+1)}{\cancel{n^3}\left(1-\frac{20}{n^3}\right)} = \frac{0^+}{1} = 0$$

Logo, w_n tende para 0

e) $\lim \sum_{k=1}^{n} \frac{n+\cos(k\pi)}{3n^2+k}$ Seja $w_n = \sum_{k=1}^{n} \frac{n+\cos(k\pi)}{3n^2+k}$

Sendo $u_n \leq w_n \leq v_n$, se $\lim u_n = \lim v_n$, então, $\lim w_n$ será igual a $\lim u_n$

$$u_n = \frac{n-1}{3n^2+n} \times n = \frac{n^2-n}{3n^2+n} = \frac{\cancel{n^2}\left(1-\frac{1}{n}\right)}{\cancel{n^2}\left(3+\frac{1}{n}\right)} \rightarrow \frac{1-0^+}{3+0^+} = \frac{1}{3}$$

$$v_n = \frac{n+1}{3n^2+1} \times n = \frac{n^2+n}{3n^2+1} = \frac{\cancel{n^2}\left(1+\frac{1}{n}\right)}{\cancel{n^2}\left(3+\frac{1}{n}\right)} \rightarrow \frac{1+0^+}{3+0^+} = \frac{1}{3}$$

Logo w_n tende para $\frac{1}{3}$

8)

a) $\lim \dfrac{n\cos^2(\alpha n)}{n^4+1} = \lim \dfrac{\cancel{n^4}\left(\overset{0^+}{\dfrac{1}{n^3}}\cos^2(\alpha n)\right)}{\cancel{n^4}\left(1+\underset{0^+}{\dfrac{1}{n^4}}\right)} = \dfrac{0^+}{1+0^+} = 0$

logo $\dfrac{n\cos^2(\alpha n)}{n^4+1}$ tende para 0

b) $\lim \dfrac{n\,\mathrm{sen}(n)}{2^n\sqrt{5n^3+1}} = \lim \dfrac{\cancel{n}\,\mathrm{sen}(n)}{2^n\,\cancel{n}\sqrt{5n+\frac{1}{n^2}}} = 0$

logo $\dfrac{n\,\mathrm{sen}(n)}{2^n\sqrt{5n^3+1}}$

c) $\lim \dfrac{n^2+3}{n\sqrt{n^3+2}} \times \cos(\sqrt{n^3+2}) = \lim \dfrac{\cancel{n^2}\left(1+\overset{0^+}{\dfrac{3}{n^2}}\right)}{\cancel{n^2}\sqrt{n+\underset{0^+}{\dfrac{2}{n^2}}}} \times \cos(\sqrt{n^3+2}) =$

$= 0$

d) $\lim n^2 \log\left(1+\frac{1}{n^2}\right) = \lim \log\left(1+\frac{1}{n^2}\right)^{n^2} = \log e = 1$

e) $\lim \left(\dfrac{(n+1)^{n+2}}{(n+2)^{n+1}} - \dfrac{n}{3}\right) \times \mathrm{sen}\left(\dfrac{1}{n}\right) =$

$= \lim \left(\dfrac{(n+1)^{n+2}}{\dfrac{(n+2)^{n+2}}{(n+2)}} - \dfrac{n}{3}\right) \times \mathrm{sen}\left(\dfrac{1}{n}\right) =$

(i) $n+1 \mid n+2$
$-n-2 \quad 1$
$0 - 1$

$= \lim \left((n+2)\left(\dfrac{n+1}{n+2}\right)^{n+2} - \dfrac{n}{3}\right) \times \mathrm{sen}\left(\dfrac{1}{n}\right) =$

$\underset{(i)}{=} \lim \left((n+2)\underbrace{\left(1+\dfrac{-1}{n+2}\right)^{n+2}}_{e^{-1}} - \dfrac{n}{3}\right) \times \mathrm{sen}\left(\dfrac{1}{n}\right) =$

$= \lim \left(\mathrm{sen}\left(\frac{1}{n}\right)(n+2)\left(1+\dfrac{-1}{n+2}\right)^{n+2} - \mathrm{sen}\left(\frac{1}{n}\right)\dfrac{n}{3}\right)$

$= \lim \left(\left(n\,\mathrm{sen}\left(\frac{1}{n}\right) + 2\,\mathrm{sen}\left(\frac{1}{n}\right)\right)\left(1+\dfrac{-1}{n+2}\right)^{n+2} - n\,\mathrm{sen}\left(\frac{1}{n}\right)\dfrac{1}{3}\right)$

$= \lim \left(\left(\overset{1}{\cancel{\dfrac{\mathrm{sen}\left(\frac{1}{n}\right)}{\frac{1}{n}}}} + 2\,\overset{0}{\cancel{\mathrm{sen}\left(\frac{1}{n}\right)}}\right)\underbrace{\left(1+\dfrac{-1}{n+2}\right)^{n+2}}_{e^{-1}} - \overset{1}{\cancel{\dfrac{\mathrm{sen}\left(\frac{1}{n}\right)}{\frac{1}{n}}}} \times \dfrac{1}{3}\right)$

$= e^{-1} - \dfrac{1}{3} = \dfrac{1}{e} - \dfrac{1}{3}$

logo $\dfrac{(n+1)^{n+2}}{(n+2)^{n+1}} - \dfrac{n}{3} \times \mathrm{sen}\left(\frac{1}{n}\right)$ tende para $\dfrac{1}{e} - \dfrac{1}{3}$

f) $\lim \frac{1}{n}\sqrt[n]{n!} =$ Se $u_n>0, \forall n \in \mathbb{N}$ e $\frac{u_{n+1}}{u_n} \to b$, então $\sqrt[n]{u_n} \to b$

$= \lim \frac{\sqrt[n]{n!}}{\sqrt[n]{n^n}} = \lim \sqrt[n]{\frac{n!}{n^n}}$

$\frac{u_{n+1}}{u_n} = \frac{\frac{(n+1)!}{(n+1)^{(n+1)}}}{\frac{n!}{n^n}} = \frac{(n+1)!\, n^n}{(n+1)^{(n+1)}\, n!} = \frac{\cancel{(n+1)}\cancel{n!}\ n^n}{\cancel{n!}(n+1)^n \times \cancel{(n+1)}} =$

$= \frac{n^n}{(n+1)^n} = \left(\frac{n}{n+1}\right)^n \overset{(i)}{=} \left(1+\frac{-1}{n+1}\right)^n =$

(i) $\begin{array}{r|l} n & n+1 \\ -n-1 & 1 \\ \hline 0-1 & \end{array}$

$= \left(\left(1+\frac{-1}{n+1}\right)^{n+1}\right)^{\frac{n}{n+1}} = \left(\left(1+\frac{-1}{n+1}\right)^{n+1}\right)^{\frac{\cancel{n}1}{\cancel{n}(1+\frac{1}{n})}} \to e^{-1} = \frac{1}{e}$

Logo, $\frac{1}{n}\sqrt[n]{n!}$ tende para $\frac{1}{e}$

g) $\lim \frac{1}{2n+2}\sqrt[n]{(2n+2)!} = \lim \frac{\sqrt[n]{(2n+2)!}}{\sqrt[n]{(2n+2)^n}} = \lim \sqrt[n]{\frac{(2n+2)(2n+1)!}{(2n+2)^n}} =$

$= \lim \sqrt[n]{(2n+2)^{1-n}\,(2n+1)!}$ Se $u_n>0, \forall n \in \mathbb{N}$ e $\frac{u_{n+1}}{u_n} \to b$, então $\sqrt[n]{u_n} \to b$

$\frac{u_{n+1}}{u_n} = \frac{(2n+4)^{-n}\,(2n+3)!}{(2n+2)^{1-n}\,(2n+1)!} = \frac{(2n+2)^{n\cancel{-1}}\,(2n+3)\cancel{(2n+2)}\cancel{(2n+1)!}}{(2n+4)^n\ \cancel{(2n+1)!}} =$

$= \frac{(2n+2)^n\,(2n+3)}{(2n+4)^n} = \left(\frac{2n+2}{2n+4}\right)^n (2n+3) =$

(i) $\begin{array}{r|l} 2n+2 & 2n+4 \\ -2n-4 & 1 \\ \hline 0-2 & \end{array}$

$\overset{(i)}{=} \left(1+\frac{-2}{2n+4}\right)^n (2n+3) = \left(1+\frac{-1}{n+2}\right)^n (2n+3)$

$= \left(\left(1+\frac{-1}{n+2}\right)^{n+2}\right)^{\frac{n}{n+2}} (2n+3) = \left(\left(1+\frac{-1}{n+2}\right)^{n+2}\right)^{\frac{\cancel{n}(1)}{\cancel{n}(1+\frac{2}{n})}} (2n+3) \to e^{-1} \times +\infty = +\infty$

Logo, $\frac{1}{2n+2}\sqrt[n]{(2n+2)!}$ tende para $+\infty$

h) $\lim \sqrt[n]{\frac{n^2\cos(\frac{1}{n})}{n!}}$ Se $u_n>0, \forall n \in \mathbb{N}$ e $\frac{u_{n+1}}{u_n} \to b$, então $\sqrt[n]{u_n} \to b$

$\frac{u_{n+1}}{u_n} = \frac{\frac{(n+1)^2\cos\left(\frac{1}{n+1}\right)}{(n+1)!}}{\frac{n^2\cos(\frac{1}{n})}{n!}} = \frac{(n+1)^2\cos\left(\frac{1}{n+1}\right)\cancel{n!}}{(n+1)\,n^2\cos(\frac{1}{n})\cancel{n!}} =$

$= \frac{(n+1)\cos\left(\frac{1}{n+1}\right)}{n^2\cos(\frac{1}{n})} = \frac{\cancel{n^2}\overbrace{\left(\frac{1}{n}+\frac{1}{n^2}\right)}^{\to 0^+}\overbrace{\cos\left(\frac{1}{n+1}\right)}^{\to 1}}{\cancel{n^2}\underbrace{\cos\left(\frac{1}{n}\right)}_{1}} =$

$= 0 \times \frac{1}{1} = 0$

Logo, $\sqrt[n]{\frac{n^2\cos(\frac{1}{n})}{n!}}$ tende para 0

i) $\lim \sqrt[n]{(n+1)! - n!}$ Se $u_n > 0, \forall n \in \mathbb{N}$ e $\frac{u_{n+1}}{u_n} \to b$, então $\sqrt[n]{u_n} \to b$

$$\frac{u_{n+1}}{u_n} = \frac{(n+2)! - (n+1)!}{(n+1)! - n!} = \frac{\cancel{n!}\left((n+2)(n+1) - (n+1)\right)}{\cancel{n!}\left((n+1) - 1\right)} = \frac{n^2 \cancel{+n} + 2n + 2 \cancel{-n} - 1}{n} =$$

$$= \frac{n^2 + 2n + 1}{n} = \frac{\cancel{n^2}\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)}{\cancel{n^2}\left(\frac{1}{n}\right)} \to \frac{1}{0^+} = +\infty$$

Logo, u_n tende para $+\infty$

j) $\lim \sqrt[n]{n! \left(\frac{2}{n}\right)^n}$ Se $u_n > 0, \forall n \in \mathbb{N}$ e $\frac{u_{n+1}}{u_n} \to b$, então $\sqrt[n]{u_n} \to b$

$$\frac{u_{n+1}}{u_n} = \frac{(n+1)! \left(\frac{2}{n+1}\right)^{n+1}}{n! \left(\frac{2}{n}\right)^n} = \frac{(n+1)\cancel{n!}\left(\frac{2}{n+1}\right)^{n+1}}{\cancel{n!}\left(\frac{2}{n}\right)^n} = \frac{\cancel{(n+1)} \frac{2^{n+1}}{(n+1)^{\cancel{n+1}}}}{\frac{2^n}{n^n}} =$$

$$= \frac{2^{n+1} \times n^n}{2^n \times (n+1)^n} = 2 \times \left(\frac{n}{n+1}\right)^n \underset{(i)}{=} 2 \times \left(\left(1 + \frac{-1}{n+1}\right)^{n+1}\right)^{\frac{n}{n+1}} =$$

(i) $n \quad | \, n+1$
$-n-1 \quad 1$
$0 - 1$

$$= 2 \times \left(\left(1 + \frac{-1}{n+1}\right)^{n+1}\right)^{\frac{\cancel{n}(1)}{\cancel{n}(1+\frac{1}{n})}} = 2 \times e^{-1} = \frac{2}{e}$$

Logo, u_n tende para $\frac{2}{e}$

9)

a) $$\frac{(-1)^n}{n} + \frac{1+(-1)^n}{2} = \begin{cases} \frac{1}{n} + \frac{1+1}{2} & \text{se } n \text{ par} \\ -\frac{1}{n} + \frac{1-1}{2} & \text{se } n \text{ ímpar} \end{cases} = \begin{cases} \frac{1}{n} + 1 & \text{se } n \text{ par} \\ -\frac{1}{n} & \text{se } n \text{ ímpar} \end{cases}$$

$$\frac{1}{n+1} \cancel{+1} - \frac{1}{n} \cancel{-1} = \frac{1}{n+1} - \frac{1}{n} = \frac{\cancel{n} \cancel{-n} - 1}{n(n+1)} = -\frac{1}{n(n+1)} < 0$$

Logo $\frac{1}{n} + 1$ é decrescente

$\inf\left(\frac{1}{n} + 1\right) = \lim\left(\frac{1}{n} + 1\right) = 1$

$\sup\left(\frac{1}{n} + 1\right) = \max\left(\frac{1}{n} + 1\right) = \frac{1}{2} + 1 = \frac{3}{2}$

$$-\frac{1}{n+1} + \frac{1}{n} = \frac{1}{n} - \frac{1}{n+1} = \frac{\cancel{n} + 1 \cancel{-n}}{n(n+1)} = \frac{1}{n(n+1)} > 0$$

Logo $-\frac{1}{n}$ é crescente

$\sup\left(-\frac{1}{n}\right) = \lim\left(-\frac{1}{n}\right) = 0$

$\inf\left(-\frac{1}{n}\right) = \min\left(-\frac{1}{n}\right) = -1$

$\sup\left(\frac{(-1)^n}{n} + \frac{1+(-1)^n}{2}\right) = \max\left\{0, \frac{3}{2}\right\} = \frac{3}{2}$

$\inf\left(\frac{(-1)^n}{n} + \frac{1+(-1)^n}{2}\right) = \min\{-1, 1\} = -1$

b) $\frac{n}{n+1} - \frac{n+1}{n} = \frac{n^2-(n+1)^2}{n^2+n} = \frac{\cancel{n^2} - \cancel{n^2} - 2n - 1}{n^2+n} = \frac{-2n-1}{n^2+n} = -\frac{2n+1}{n^2+1} > 0$

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Logo $\frac{n}{n+1} - \frac{n+1}{n}$ é crescente

$\inf\left(-\frac{2n+1}{n^2+1}\right) = \min\left(-\frac{2n+1}{n^2+1}\right) = -\frac{2\times1+1}{1^2+1} = -\frac{3}{2}$

$\sup\left(-\frac{2n+1}{n^2+1}\right) = \lim\left(-\frac{2n+1}{n^2+1}\right) = \lim\left(-\frac{\cancel{n^2}\left(\frac{2}{n}+\frac{1}{n^2}\right)}{\cancel{n^2}\left(1+\frac{1}{n^2}\right)}\right) = -\frac{0^+}{1} = 0$

10.
a) $\frac{1}{n^2}\cos\left(\frac{n\pi}{10}\right) + \left(\cos\left(\frac{n\pi}{2}\right)\right)^n$

$\lim\left(\frac{1}{n^2}\cos\left(\frac{n\pi}{10}\right)\right) + \lim\left(\cos\left(\frac{n\pi}{2}\right)\right)^n \rightarrow$ como $\lim \frac{1}{n^2} \to 0$ e a função cos é uma função periódica, então: $\lim\left(\frac{1}{n^2}\cos\left(\frac{n\pi}{10}\right)\right) = 0$

$= 0 + \lim\left(\cos\left(\frac{n\pi}{2}\right)\right)^n$

$\left(\cos\left(\frac{n\pi}{2}\right)\right)^n = \begin{cases} 1 & \text{se } n=4k, k\in\mathbb{N} \\ 0 & \text{se } n=4k+1, k\in\mathbb{N} \\ (-1)^n & \text{se } n=4k+2, k\in\mathbb{N} \\ 0 & \text{se } n=4k+3, k\in\mathbb{N} \end{cases} = \begin{cases} 1 & \text{se } n \text{ par} \\ 0 & \text{se } n \text{ ímpar} \end{cases}$

Logo, 0 é o limite inferior e 1 é o limite superior

b) $\frac{(-1)^n n^2+3}{n+1} = \begin{cases} \frac{n^2+3}{n+1} & \text{se } n \text{ par} \\ \frac{-n^2+3}{n+1} & \text{se } n \text{ ímpar} \end{cases}$

$\frac{n^2+3}{n+1} > 0$, logo é crescente

$\inf\left(\frac{n^2+3}{n+1}\right) = \min\left(\frac{n^2+3}{n+1}\right) = \frac{1^2+3}{1+1} = \frac{4}{2} = 2$

$\sup\left(\frac{n^2+3}{n+1}\right) = \lim\left(\frac{n^2+3}{n+1}\right) = \lim\left(\frac{\cancel{n^2}\left(1+\frac{3}{n^2}\right)}{\cancel{n^2}\left(\frac{1}{n}+\frac{1}{n^2}\right)}\right) = \frac{1}{0^+} = +\infty$

$\frac{-n^2+3}{n+1} < 0$, logo é decrescente

$\inf\left(\frac{-n^2+3}{n+1}\right) = \lim\left(\frac{-n^2+3}{n+1}\right) = \lim\left(\frac{\cancel{n^2}\left(-1+\frac{3}{n^2}\right)}{\cancel{n^2}\left(\frac{1}{n}+\frac{1}{n^2}\right)}\right) = -\frac{1}{0^+} = -\infty$

$\sup\left(\frac{-n^2+3}{n+1}\right) = \max\left(\frac{-n^2+3}{n+1}\right) = \frac{-1^2+3}{1+1} = \frac{2}{2} = 1$

Logo, $+\infty$ é o limite inferior e $+\infty$ é o limite superior

c) $\frac{((-1)^{n+3}-(-1)^n)n^3+2}{3n+1} = \begin{cases} \frac{(-1-1)n^3+2}{3n+1} & \text{se n par} \\ \frac{(1+1)n^3+2}{3n+1} & \text{se n ímpar} \end{cases} =$

$= \begin{cases} \frac{-2n^3+2}{3n+1} & \text{se n par} \quad 0 \quad -\infty \\ \frac{2n^3+2}{3n+1} & \text{se n ímpar} \quad \frac{4}{4}=1 \quad +\infty \end{cases}$

$\frac{-2n^3+2}{3n+1} < 0$, logo é decrescente

$\inf\left(\frac{-2n^3+2}{3n+1}\right) = \max\left(\frac{-2n^3+2}{3n+1}\right) = \frac{-2\times1^3+2}{3\times1+1} = \frac{0}{4} = 0^+$

$\sup\left(\frac{-2n^3+2}{3n+1}\right) = \lim\left(\frac{-2n^3+2}{3n+1}\right) = \lim\left(\frac{\cancel{n^3}(-2+\frac{2}{n^3})}{\cancel{n^3}(\frac{3}{n^2}+\frac{1}{n^3})}\right) = -\frac{2}{0^+} = -\infty$

$\frac{2n^3+2}{3n+1} > 0$, logo é crescente

$\sup\left(\frac{2n^3+2}{3n+1}\right) = \lim\left(\frac{2n^3+2}{3n+1}\right) = \lim\left(\frac{\cancel{n^3}(2+\frac{2}{n^3})}{\cancel{n^3}(\frac{3}{n^2}+\frac{1}{n^3})}\right) = \frac{2}{0^+} = +\infty$

$\inf\left(\frac{2n^3+2}{3n+1}\right) = \min\left(\frac{2n^3+2}{3n+1}\right) = \frac{2\times1^3+2}{3\times1+1} = \frac{4}{4} = 1$

Logo, $-\infty$ é o limite inferior e $+\infty$ é o limite superior

(11) Como $u_{n+1} > 0$, então $u_n > 0$, como $u_1 = 1$ e $u_n < \frac{3}{2}$, então a sucessão é limitada, logo é convergente

$\lim u_{n+1} = \lim u_n = a$

$a = \frac{a+3}{3} \Leftrightarrow 3a = a+3 \Leftrightarrow 2a = 3 \Leftrightarrow a = \frac{3}{2}$

Logo, $\lim u_n = \frac{3}{2}$

(12) Como $u_{n+1} > 0$, então $u_n > 0$, com u_1 0 e $u_n < 3$, então a sucessão é limitada, logo é convergente

$\lim u_{n+1} = \lim u_n = a$

$a = \sqrt{a+6} \Leftrightarrow a^2 = a+6 \Leftrightarrow a^2 - a - 6 = 0 \underset{(i)}{\Leftrightarrow}$

$\Leftrightarrow a = 3$

Logo, $\lim u_n = 3$

(i) $a = \frac{1 \pm \sqrt{1-4\times1\times(-6)}}{2}$

$\Leftrightarrow a = \frac{1\pm\sqrt{25}}{2}$

$\Leftrightarrow a = -2 \vee \underset{0 \leq a < 3}{\underline{a = 3}}$

13.

a) $a_n = \begin{cases} \frac{n+1}{n} & \text{se } n \text{ par} \\ \frac{n}{n+1} & \text{se } n \text{ ímpar} \end{cases}$

Termos de $a_n = \{\frac{1}{2}, \frac{3}{2}, \frac{3}{4}, \frac{5}{4}, \frac{5}{6}, \frac{7}{6}, \dots\}$

A sucessão é não monótona

b) É limitada, majorante: $\frac{3}{2}$, minorante: $\frac{1}{2}$

c) $\lim\left(\frac{n+1}{n}\right) = \lim\left(\frac{\cancel{n}(1+\overbrace{\frac{1}{n}}^{0^+})}{\cancel{n}}\right) = \frac{1+0^+}{1} = 1^+$

$\lim\left(\frac{n}{n+1}\right) = \lim\left(\frac{\cancel{n}}{\cancel{n}(1+\underbrace{\frac{1}{n}}_{0^+})}\right) = \frac{1}{1+0^+} = 1^-$

Logo a sucessão é convergente, tendo como limite 1